Verified Enclosure of Invariant Manifolds of Planar Diffeomorphisms and Application to Homoclinic Phenomena

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Interesting questions in the study of the dynamics of planar diffeomorphisms like the Henon map involve homoclinic phenomena, topological entropy and strange attractors. Attempts to answer these questions generally require knowledge about the stable and unstable manifolds, which in the typical case in the plane are smooth curves.
We present a method that combines Taylor Model arithmetic with normal-form-transformation based techniques to find highly accurate Taylor Model enclosures of the invariant curves near hyperbolic fixed points. Successive iteration of these local manifold enclosures yields similarly accurate enclosures of significant pieces of the global invariant manifolds.
An interesting application is the automated computation of quite large numbers of homoclinic points of a given system, as well as their mapping properties. There is evidence that this information will be sufficient to automatically compute good lower bounds for the topological entropy of the system.