Optimizing the adiabatic buncher and phase-energy rotator for neutrino factories

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Abstract

In the US scenario for a Neutrino Factory presented in "A feasibility study of a neutrino source based on a muon storage ring", N. Holtkamp (Ed.), D. Finley (Ed.), Fermilab, April 15th, 2000), a large percentage of the cost is related to an induction linac for phase-energy rotation and bunching of the muon beam collected after the production target and decay channel. A more cost-effective adiabatic buncher and phase-energy rotator has been proposed to replace the induction linac system (D. Neuffer, A. Van Ginneken, High-frequency bunching and ($\phi - \delta E$) rotation for a muon source, Proceedings of the 2001 Particle Accelerators Conference, Chicago, 2001, p. 2029). The new method uses consecutive RF cavities with differing frequencies. The frequencies are changed to enable bunching and phase-energy rotation. In this paper, the theoretical concept is developed and demonstrated with simulation results obtained with the map code *COSY Infinity* (http://cosy.pa.msu.edu). An optimization strategy is also explored.
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1. Introduction

In various scenarios for muon-based accelerator projects such as a Muon Collider or a Neutrino Factory (Fig. 1), phase-energy ($\phi - \delta E$) rotation is used in the μ beam exiting the π production and decay channel, because this beam has not just a relatively small initial phase spread, but an energy spread that is much larger than the device acceptance [1–3]. In this process the beam is first allowed to lengthen and then the radio-frequency (RF) system is used to reduce the energy spread (by decelerating the high-energy "head" of the bunch and accelerating the low-

energy "tail", so that the beam "rotates" in phase-energy space). The resulting beam has the energy spread reduced to a level where the majority of the beam particles is captured by a subsequent bunching and/or cooling system. The phase-energy rotation region is outlined by rectangle in Fig. 1.

The difficulty with the previously proposed ($\phi - \delta E$) rotation systems is that they require either very low-frequency RF, or an induction linac, matched to the elongated bunch length of the ($\phi - \delta E$) rotated system. This long-wavelength (or long rise-time) acceleration system would require new technology development and considerable expense. In this paper we present an approach which uses high-frequency RF systems for bunching the beam and reducing its overall energy spread [4]. With this approach it is possible to produce a particle distribution similar to that obtained in the induction linac and RF buncher system proposed in Study 2 [1–3], except that this

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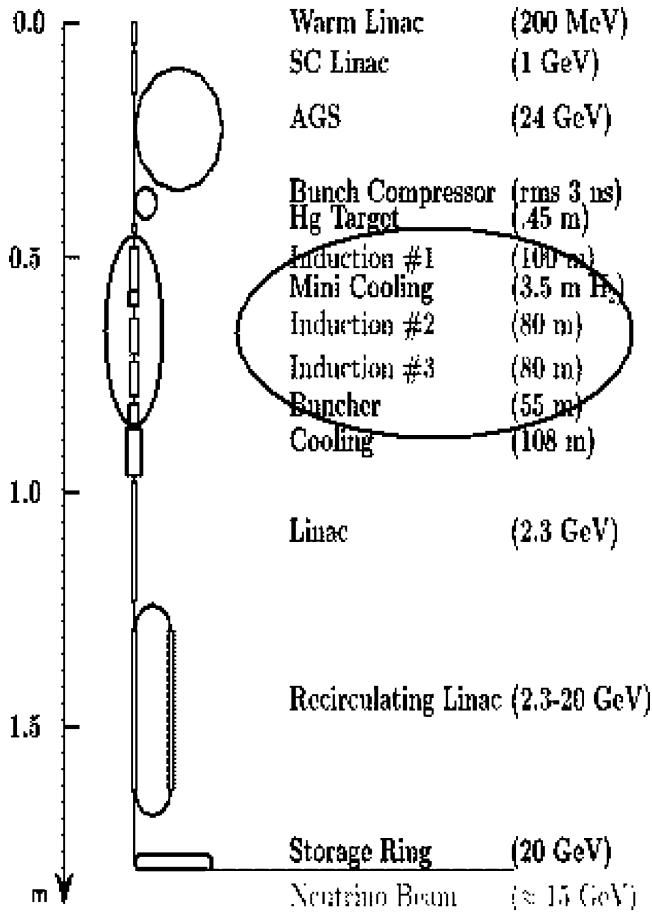
Fig. 1. Neutrino Factory schematics as proposed in Study 2 with outlined phase-energy rotation section.

system simultaneously captures both μ^+ and μ^-. The concept, key parameters, example simulations, and an optimization strategy based on the control theory approach are presented.

2. Concept

The initial μ beam with a small phase spread and a large energy spread from a $\pi \to \mu$ production target is allowed to drift in a solenoidal field. The *drift* section is followed by *adiabatic buncher* where beam is formed into a string of bunches and *phase-energy rotator* where its overall energy spread is reduced. Both buncher and rotator consists of RF cavities within a solenoidal (transversely focusing) field (Fig. 2).

To bunch particles we choose some particle to be the main central particle of the beam. We set all RF cavities parameters in such a way that this particle passes every one of them in the same phase of E field oscillations (0 in a buncher). By the virtue of the equations of motion in such a structure, particles close enough to this central one, will be formed into a stable group called "bunch". Because of the specific choice of this main central particle's phase and cavities parameters, we also have some other particles passing cavities in the same 0 phase and, by the same equations of motion, we will have bunching effect around those particles as well. In the following text we will call them "central particles" and the one chosen first "main central particle". Of course, all central particles are not real particles, they are just an idealization chosen to make equations of motion simpler.
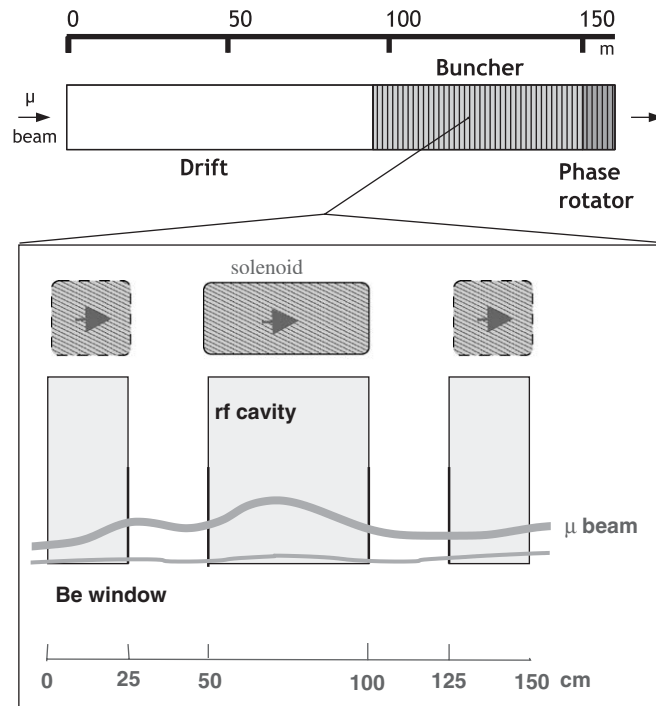

Fig. 2. Example simulation plots in (T,ct) phase space. Beam is shown after drift, buncher, rotator and cooler, respectively.

Each cavity in the buncher has its frequency set to maintain the following condition: the time of arrival difference between two central particles in a place of RF field application remains equal to a fixed integer number of RF oscillations periods so this condition holds true as the beam propagates through the buncher. As we set E field phase to be 0 for the main central particle, for other central particles it is also 0 so they gain no energy in each cavity and their energies stay constant through the buncher. We keep the final system frequency fixed because of matching into 201.25 MHz cooling and/or accelerating sections, so, using these two conditions we can find a frequency for each RF cavity as a function of its position in a buncher.

In the buncher, the RF gradient is increased as length increases. The goal here is to perform an *adiabatic capture*, in which the beam within each bunch is compressed in phase so as to be concentrated near the central particle. From these considerations we obtain the following relations for the lattice parameters and central particles of the bunches:

$$\Delta t = t_{\mathrm{n}} - t_{\mathrm{c}} = z\left(\frac{1}{v_{\mathrm{n}}} - \frac{1}{v_{\mathrm{c}}}\right) = nT_{\mathrm{RF}} = n\frac{\lambda_{\mathrm{RF}}}{c}, \quad n \in \mathbb{Z} \quad (1)$$

$$\delta\left(\frac{1}{\beta}\right) = \left(\frac{1}{\beta_1} - \frac{1}{\beta_{\mathrm{c}}}\right) = \frac{\bar{\lambda}}{\bar{L}} \quad (2)$$

from which follows

$$\frac{1}{\beta_{\mathrm{n}}} = \frac{1}{\beta_{\mathrm{c}}} + n\delta\left(\frac{1}{\beta}\right) \Rightarrow T_{\mathrm{n}}, \quad n \in \mathbb{Z} \quad (3)$$

$$\lambda_{\mathrm{RF}}(z) = z \cdot \delta\left(\frac{1}{\beta}\right) \Rightarrow v_{\mathrm{RF}} = \frac{c}{z \cdot \delta\left(\frac{1}{\beta}\right)} \quad (4)$$

$$V_{\mathrm{RF}}(z) = B\frac{(z - z_{\mathrm{D}})}{L} + C\frac{(z - z_{\mathrm{D}})^2}{L} \quad (5)$$

where n — denote the number of the bunch counted from the main central particles's one, T_{RF} — period of RF field oscillations, z — longitudinal coordinate counted from the beginning of the drift, β_{c} and β_{n} — normalized central particle and n-th reference particle's velocities, t_{c} and t_{n} — time of arrival of main central and n-th central particle (main central particle has $n = 0$); λ_{RF}, v_{RF} and V_{RF} — wavelength, frequency and gradient of the electric field in the cavity, z_{D} — longitudinal coordinate of the beginning of the buncher (equals to the drift length); $\bar{\lambda}$ and $\bar{L}$ — wavelength of electric field and the longitudinal coordinate of the last RF in buncher; c — speed of light, B and C — positive constants, defining RF gradients in a buncher. Note that, since each of the bunches is centered at different energy, they all have different longitudinal oscillation frequencies, and a simultaneously matched compression for all bunches is not possible. Instead a quasi-adiabatic capture resulting in an approximate bunch length minimization in each bunch is attempted.

Following the buncher is the so-called $(\phi - \delta E)$ *vernier rotation system* in which the RF frequency is almost fixed to the matched value at the end of the buncher and the RF voltage is constant. In this system the energies of the central particles of the low-energy bunches increase, while those of the high-energy bunches decrease. So the whole energy spread reduces to the point where beam is a string of similar-energy bunches and it could be captured into the ~200 MHz ionization cooling system matched to the central energy of the beam.

Let us describe the rotator parameters calculation in more detail. At the end of the buncher we choose a second central particle kept N RF periods from main central one along the buncher and the *vernier offset* δ. We then keep this second central particle at $(N + \delta)\lambda_{\mathrm{RF}}$ wavelengths from the main one through the rotator. So now it passes all RFs in a constant accelerating phase ϕ_{N} having constant energy gain ΔT_{N}, and after $|T_{\mathrm{c}} - T_{\mathrm{N}}|/\Delta T_{\mathrm{N}}$ cavities, energies of the central particle and the chosen second reference one will be nearly equal. This process also aligns the energies of other reference particles and their bunches, hence at the end we have the beam rotated in $(\phi - \delta E)$ space with significantly reduced energy spread.

Example simulation of this process in 1D was developed in a Pascal code [5,6]. We take the main central particle's energy to be 125 MeV, beam's energy spread to be ±50 MeV and $(\phi - \delta E)$ coordinates distribution to be Gaussian. We arbitrarily set the initial drift length to 90 m and define buncher to consist of 60 pillbox cavities 1 m long each. With these numbers we get $\delta(1/\beta) = 1.5/150 = 0.01$, so, by plugging this values into Eq. (4) we obtain the RF frequency at the beginning of the buncher section ~333 MHz, and at the end ~200 Mhz (we match into a 201.25 Mhz cooling system). We choose the RF gradient to be quadratically increasing from 0 to 4.8 MV/m along the buncher, so, from Eq. (5) it follows that

$$V_{\mathrm{RF}}(z) = 4.8\frac{(z - z_{\mathrm{D}})^2}{(\bar{L} - z_{\mathrm{D}})^2}\,\mathrm{MV/m}. \quad (6)$$

In rotator we choose $N = 10 \Rightarrow T_{\mathrm{N}} \approx 77.28$ MeV; $\delta = 0.1 \Rightarrow \phi_{\mathrm{N}} = 36°$ and the RF gradient to be 10 MV/m, so, after $\frac{1}{4}$ synchrotron oscillation (~8.4 m), the central energies spread becomes nearly 0. Coordinates of the particles in $(\phi - \delta E)$ phase space through the structure are shown in 2.

3. Problem description and key parameters/controls

The concept is defined in the previous section, but there are many variations in the structure parameters because of the minimal cost constraint, different possible final RF frequencies, reduced number of RF frequencies and gradients, etc., and in the final beam properties: shorter/longer bunch trains, constraints on the number of muons captured, desired central energy, etc. Matching into the accelerating/cooling structures following the buncher-rotator system and the transverse beam dynamics should

also be considered. The problem is in finding specific optimal parameters of the beam and/or the structure under imposed constraints.

In our example we use 60 RF cavities, each with different gradient and frequency, which is definitely makes structure too expensive to be built. Structure consisted of 10 cavities would be much more satisfactory in a cost sense. We could also try to combine the buncher and the $(\phi - \delta E)$ rotator into one structure for simultaneous bunching and rotation to reduce cost of the system. As could be seen from the concept and the relations given above, the control parameters of the structure are:

(1) *Drift*: the length of the section L_{D}. Future studies, which include transverse motion, must also consider the apertures and focusing fields (this study uses fixed-field solenoids for transverse focusing). These focusing parameters are also critical for system performance.
(2) *Buncher*: the length of the section L_{B}, RF voltages V_{B}^{i}, $i = \overline{1, n_{\mathrm{RFs}}}$ or initial and final voltage and the law of voltage increase (linear, quadratic, etc.). Final frequency is usually strictly specified by the cooling/accelerating subsections of the accelerator, but could also be varied to find optimum.
(3) $(\phi - \delta E)$ *Rotator*: the length $L_{\phi \mathrm{R}}$, RF voltage $V_{\phi \mathrm{R}}$ of the phase-energy rotation section, number N of RF field oscillation periods between chosen second central particle and the main central particle (with $n = 0$), and the vernier parameter δ. Also the kinetic energy T_{c} of the main central particle could be changed (usually we take T_{c} to be the peak of energy distribution of beam's particles).

4. Simulations

Advanced particles dynamics simulation program was written in the *COSY Infinity* code [7,8] which uses a map-based approach of beam dynamics calculation. The dynamics is described in terms of a high-order Taylor expansion of the flow, i.e. the relationship that connects final particle coordinates to initial coordinates via

$$\vec{z}_f = \mathcal{M}(\vec{z}_i) \tag{7}$$

where the flow (also called "map") $\mathcal{M}$ is determined by either automatic differentiation of numerical integration algorithms, or by dedicated tools determining the flow of partial differential equations based on differential algebraic techniques. Depending on the complexity of the task, different orders of expansion from 1 to 15 are needed, and in our case, the necessary order is high ($\geqslant 7$). Apart from the powerful abilities to calculate high-order dynamics properties, *COSY Infinity* has its own programming language, which allows the construction of complicated optimization scenarios, it provides user with powerful DA (*differential algebra*) framework, and it has large built-in

library of standard accelerator elements, so it fits well to our needs.

Because the beam has a very large energy spread and *COSY Infinity* calculates dynamics with the use of Taylor expansions on particle coordinates, a division of the initial coordinates domain into sub-domains with small energy coordinate range is required before tracking. The natural way of doing this is to divide the set of coordinates by the number of bunches in a beam. This division makes relative coordinates small enough to have Taylor expansions with acceptably small remainders at a reasonable order (order 7 or 8 is enough). The standard RF kick approximation for a pillbox RF cavity is used, i.e. we assume particles to pass the cavity instantly so that field does not change, having constant energy increment/decrement dependent on the particle coordinates and cavity parameters. This approximation is suitable for our simulations because our particles are fast enough (200 MeV μ has $\beta \approx 0.94$) and cavities are short enough to neglect any field change during particle transition. If deemed necessary, in future simulations a more realistic RF cavity model like the one developed in Ref. [4] could be used.

In our example structure we use 50 sub-domains for more realistic purely Gaussian (larger energy spread) and "almost real" distributions from *MARS* code [9]. Simulation results for these distributions are shown in Figs. 3–5.

5. Optimization problem formulation

The problem of simulation and search for optimal parameters naturally presents itself as one of the problems of control theory in beam physics [10]. The exact definition of the problem depends on what parameters of the structure lattice and/or the beam at the end of the structure are critical to achieve. We can consider an impulse effect model:

$$\vec{x}_k = \mathcal{A}^k(\vec{x}_{k-1}, \vec{u}_k) \tag{8}$$

where $\vec{x}_k$ is a vector of coordinates in phase space after the k-th lattice element represented by the $\mathcal{A}^k$ operator and the control $\vec{u}_k$. As could be seen from comparison with Eq. (7), this method is exactly the one used in *COSY Infinity* to calculate beam dynamics. We can as well use the continuous model

$$\dot{x} = f(t, x, u), \quad t \in [0, T], \ x \in \Omega \subset \mathbb{R}^{\mathrm{n}}, \ u \in U \subset \mathbb{R}^{\mathrm{r}} \tag{9}$$

where x, u are coordinate and control vectors, respectively, t is the time of flight, and f is a continuous function, which describes particle's dynamics dependence on control functions representing the structure's parameters. On the trajectories obtained from any of these models, we define a quality functional as

$$I(u) = \int_0^T \int_{M_{t,u}} \phi(t, x_t, u(t)) \, \mathrm{d}x_t \, \mathrm{d}t + \int_{M_{T,u}} g(x_T) \, \mathrm{d}x_T \tag{10}$$

where x_t, x_T are the coordinates of the particle at the time t and at the terminal time T respectively, $M_{t,u}$, $M_{T,u}$ are the
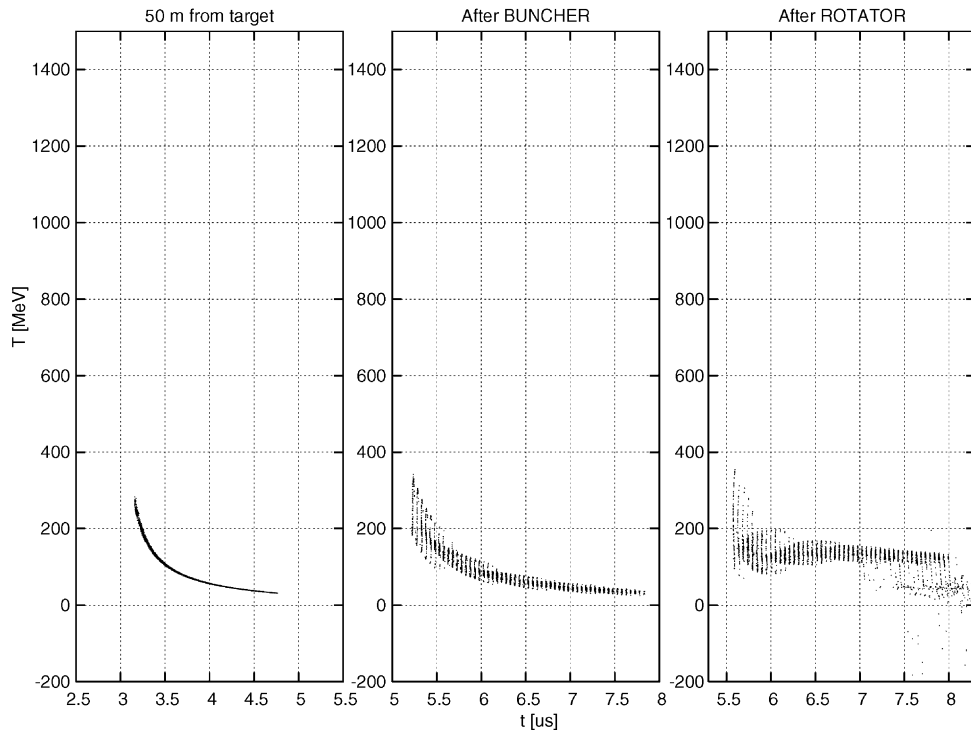
Fig. 3. *COSY Infinity* simulations plots in (T, t) phase space. Initial distribution Gaussian, $T = 125 \, \text{MeV} \pm 50 \, \text{MeV}$).
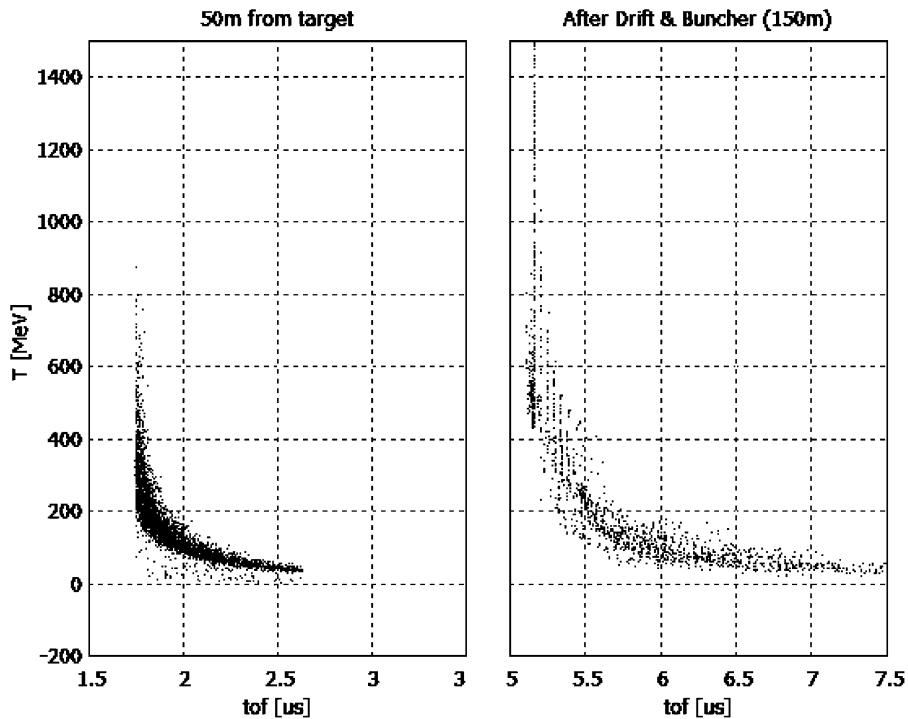

Fig. 4. *COSY Infinity* simulations plots in (T, t) phase space. Initial distribution Gaussian $T = 10 - 1000 \, \text{MeV}$.

sets of coordinates of beam particles at the time t and at the terminal time T, respectively.

Functionals of the type (10) with problem-defined functions ϕ and g allow one to evaluate any desired beam parameters throughout the whole structure with the first item and the terminal beam parameters with the second item. In search for optimal structure parameters (optimal

in a sense they make dynamics of the particles and terminal beam parameters optimal) one then needs to find control functions that brings minimum/maximum to this functional.

As a variant of such formulation applicable to our problem we consider the problem of optimal transportation of the initial coordinates $\vec{x} = (x_1, x_2)^{\text{T}}$ set $M_{0,u}$ into the
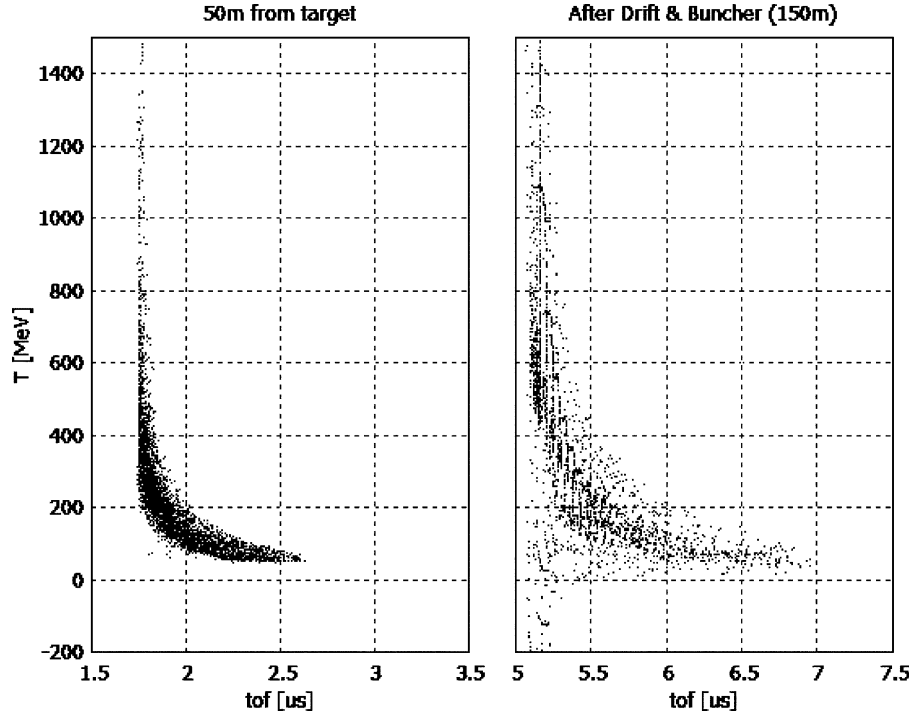
Fig. 5. *COSY Infinity* simulations plots in (T, t) phase space. "Realistic" initial distribution obtained from *MARS* code [9].
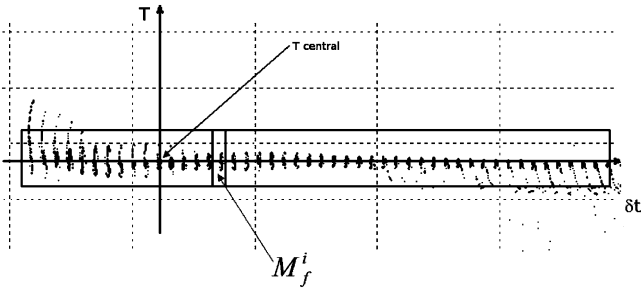

Fig. 6. Final beam structure and definition of bounding sets.

set of final coordinates $M_{T,u}$ with specified boundaries, i.e. we will base our evaluation on the terminal beam parameters. In our case $x_1 \leftrightarrow \phi$, $x_2 \leftrightarrow \delta E$. As an initial coordinate boundary we take a rectangle in phase-energy phase space, which encloses all or almost all particles of the beam. For the final boundary we may consider another rectangle with shorter length along energy coordinate as we are interested in reducing overall energy spread. But, in fact, we are also interested in enclosing the particles in each bunch to a so-called bucket area, because apart from the small energy spread we have the constraint of capturing, i.e. particles in each bunch should be in a stable area called "bucket" (in sense of equation of motion in resonant RF structure which has stable and unstable solutions). So we might divide this bounding rectangle into sub-rectangles (Fig. 6), the number of which is equal to the number of bunches in a beam, and define a penalty function to evaluate proximity of the terminal particles coordinates in each bunch to the rectangle corresponding to this

bunch $[a_1, a_2] \times [b_1, b_2]$

$$\phi_1(\vec{x}) = \begin{cases} 0, & x_1 \in [a_1, a_2] \\ k_1(x_1 - a_2)^{2q_1}, & x_1 \geqslant a_2 \\ k_1(a_1 - x_1)^{2q_1}, & x_1 \leqslant a_1 \end{cases} \qquad (11)$$

$$\phi_2(\vec{x}) = \begin{cases} 0, & x_2 \in [b_1, b_2] \\ k_2(x_2 - b_2)^{2q_2}, & x_2 \geqslant b_2 \\ k_2(b_1 - x_2)^{2q_2}, & x_2 \leqslant b_1 \end{cases} \qquad (12)$$

where k_1, k_2, q_1, q_2 are arbitrary positive weight constants for selective optimization. Then, we define the quality functional as

$$I = \int_{M_{T,u}} (C_1\phi_1(\vec{x}) + C_2\phi_2(\vec{x})) \, d\vec{x}. \qquad (13)$$

With this functional we could perform optimization using some method of functional minimization (stochastic, gradient, etc.). This optimization is the main direction of the future research.

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