Exploring the Bunching Section of the Neutrino Factory

Alexey A. Poklonskiy, David Neuffer, Martin Berz, Dmitriy A. Ovsyannikov, Alexandre D. Ovsyannikov

Abstract— **There exists now a large demand in better neutrino beams in particle physics community. Studying of such beams could reveal interesting and important properties of the observed neutrino oscillations. A high intensity source of a single flavor of neutrinos with reduced backgrounds, a known energy spectrum and intensity is needed for this research. Such intense source of neutrinos is supposed to be provided by the Neutrino Factory. A Neutrino Factory, as proposed, relies on formation and acceleration of ultra-large emittance muon beams with subsequent decay of the muons into a well-collimated, well-characterized neutrino beam. The muon beam creating section of the lattice is originally based on capturing, bunching and phase rotation in an expensive induction linac. A recently proposed different scheme bunches particles and reduces their energy spread in an array of high-frequency rf cavities whose rf frequency varies along the length of the channel. The cost reduction and simplicity of the proposed approach is extensive but is still not at its optimum. Different variations of the design paramaters leading to different properties of the beam and cost are possible. This work explores the approach, variations and develops an optimization scheme for rf parameters based on the underlying beam dynamics.**

I. Introduction

A Neutrino Factory [1], [2] facility offers an exciting option for the long-term neutrino physics program. New accelerator technologies offer the possibility of building, in the not-too-distant future, an accelerator complex to produce and capture more than 1020 muons per year [2]. It has been proposed to build a Neutrino Factory by accelerating the muons from this intense source to energies of several tens of GeV, injecting them into a storage ring which has straight sections, and exploiting the intense neutrino beams that are produced by muons decaying in these straight sections. The decays

$$\mu^- \to e^- \nu_\mu \bar{\nu}_e \,, \quad \mu^+ \to e^+ \bar{\nu}_\mu \nu_e$$

offer exciting possibilities to pursue the study of neutrino oscillations and neutrino interactions with exquisite precision. A Neutrino Factory requires an intense multi-GeV proton source capable of producing a primary proton beam with a beam power of 1.2 MW or more on target. This is the same proton source required in the medium term for Neutrino Superbeams; hence, there is a natural evolution from Superbeam experiments to Neutrino Factory experiments in the longer term. The physics case for a Neutrino Factory will depend upon results from the next round of planned neutrino oscillation experiments [3]. If there are no experimental surprises, the physics case for a Neutrino Factory will depend on the values of the oscillation parameters, the achievable sensitivity that will be demonstrated by the first generation of ν_e appearance experiments, and the nature of the second generation of basic physics questions that will emerge from the first round of results. In either case in about a decade the neutrino community may need to insert a Neutrino Factory into the global neutrino plan. The option to do this in the next 10 years will depend upon the accelerator R&D that is done during the intervening period.

In the U.S., the Neutrino Factory and Muon Collider Collaboration [4] is a collaboration of 130 scientists and engineers engaged in carrying out the accelerator R&D that is needed before a Neutrino Factory could be inserted into the global plan. Much technical progress has been made over the last few years, and the required key accelerator experiments are now in the process of being proposed and approved. In addition to the U.S. effort, there are active Neutrino Factory R&D groups in Europe and Japan, and much of the R&D is performed and organized as an international endeavor. Thus, because a Neutrino Factory is potentially the key facility for the long-term neutrino program, Neutrino Factory R&D is an important part of the present global neutrino program.

In this article we describe and demonstrate an approach to optimization of one of the cruicial stages in the current Neutrino Factory design. The use of this approach potentially leads to the structure with increased perfomance and/or reduced cost.

II. Neutrino Factory Design

In this section we describe the basic machine concepts that are used to create a Neutrino Factory facility. This facility is a secondary beam machine, that is, a production beam is used to create the secondary beam that eventually provides the neutrino flux for the detector. For a Neutrino Factory, the production beam is a high intensity proton beam of moderate energy (beams of 2.50 GeV have been considered by various groups) that impinges on a target, typically a high-Z material (e.g. Hg). The collisions between the proton beam and the target nuclei produce a secondary pion beam that quickly decays (26.0 ns) into a longer-lived (2.2 μs) muon beam. The remainder of the Neutrino Factory is used to condition the muon beam, accelerate it rapidly to the desired final energy of a few tens of GeV, and store it in a decay ring which a long straight section oriented such that decay neutrinos produced

Alexey A. Poklonskiy (poklonsk@fnal.gov), Dmitriy A. Ovsyannikov (dovs@apmath.spbu.ru), Alexandre D. Ovsyannikov (ovs@apmath.spbu.ru): Universitestkiy Prospekt, 35, Petergof, Faculty of Applied Mathematics and Control Processes, Saint-Petersburg State University, St. Petersburg, Russia, 198504; David Neuffer (neuffer@fnal.gov): MS221, Fermi National Accelerator Laboratory, PO Box 500, Batavia, IL 60510-500, USA, Martin Berz (berz@msu.edu): Department of Physics and Astronomy Michigan State University, East Lansing, MI 48824, USA

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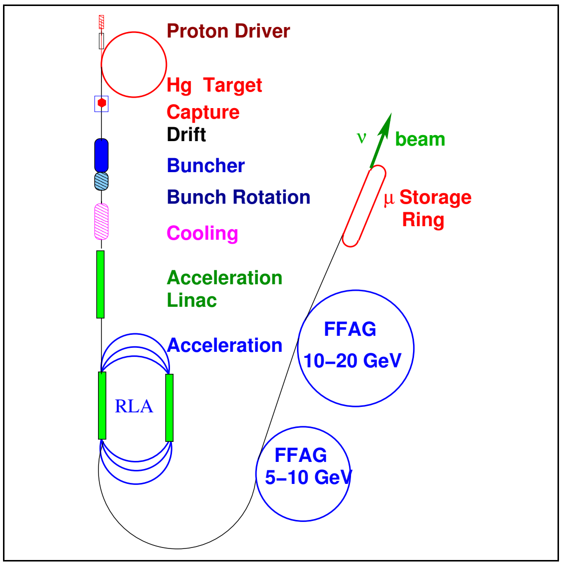

Fig. 1. Neutrino Factory Schematics (Study 2a)

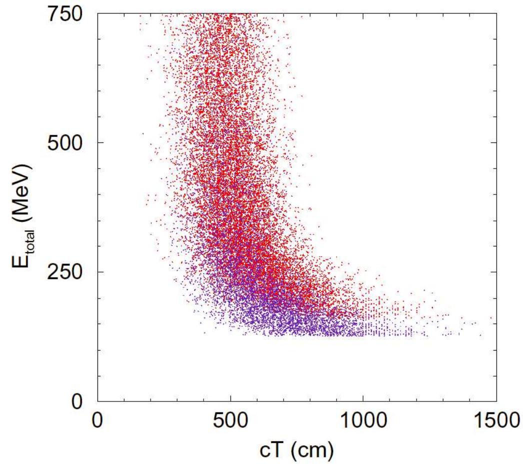

Fig. 2. Distribution of particles energies 12m from the target calculated by MARS

there will hit a detector located thousands of kilometers from the source.

The various components of a Neutrino Factory, based in part on the most recent Feasibility Study (Study 2) [5] that was carried out jointly by Brookheaven National Laboratory (BNL) and the U.S. Neutrino Factory and Muon Collider Collaboration, are described briefly below. Details of this design are based on the specific scenario of sending a neutrino beam from BNL to a detector in Carlsbad, New Mexico. More generally, however, the design exemplifies a Neutrino Factory for which two Feasibility Studies have demonstrated technical feasibility (provided the challenging component specifications are met), established a cost baseline, and established the expected range of physics performance. It is worth noting that this Neutrino Factory design could fit comfortably on the site of an existing U.S. laboratory, such as BNL or Fermi National Accelerator Laboratory (FNAL).

The main ingredients of a Neutrino Factory (Fig.1) include:

- **Proton Driver.** Provides 1.4 MW of protons on target.
- **Target and Capture.** A high-power target immersed in a 20 T superconducting solenoidal field to capture pions produced in proton-nucleus interactions. The high magnetic field at the target is smoothly tapered down to a much lower value, 1.75 T, which is then maintained through the bunching and phase rotation sections of the Neutrino Factory.
- **Bunching and Phase Rotation.** First the bunching with rf cavities of modest gradient, whose frequencies change as we proceed down the beam line is performed. After bunching the beam, another set of rf cavities, with higher gradients and again having frequencies decreasing downward the beam line, is used to rotate the beam in longitudinal phase space to reduce its energy spread.
- **Cooling.** A solenoidal focusing channel, with high-gradient 201.25 MHz rf cavities and LiH absorbers, cools the transverse normalized rms emittance from 17 mm·rad to about 7 mm·rad. This takes place at a central muon momentum of 220 MeV/c.
- **Acceleration.** A superconducting linac with solenoidal focusing is used to raise the muon beam energy to 1.5 GeV, followed by a Recirculating Linear Accelerator (RLA), arranged in a dogbone geometry, to provide a 5 GeV muon beam. Thereafter, a pair of cascaded Fixed-Field, Alternating Gradient (FFAG) rings, having combined-function doublet magnets, is used to reach 20 GeV. Additional FFAG stages could be added to reach a higher beam energy, if the physics requires this.
- **Storage Ring.** A compact racetrack-shaped superconducting storage ring in which $\approx$ 35% of the stored muons decay toward a detector located some 3000 km from the ring. Muons survive for roughly 500 turns.

III. Buncher and Phase Rotator

Pions, and the muons into which they decay, are generated in the target over a very wide range of energies (see Fig.2) , but in a short time pulse ($\approx$ 3 ns rms). To prepare the muon beam for acceleration thus requires significant conditioning including reducing energy spread anf forming the beam into the string of bunches. First, the beam is drifted to develop an energy correlation, with higher energy particles at the head and lower energy particles at the tail of the beam. Next, the long beam is separated into a number of short bunches suitable for capture and acceleration in a 201-MHz rf system. This is done with a series of rf cavities having frequencies that decrease and gradients that increase along the beam line, separated by suitably chosen drift spaces. The resultant bunch train still has a substantial energy correlation, with the higher energy bunches first and progressively lower energy bunches coming behind. The large energy tilt is then phase rotated, using additional rf cavities of decreasing frequencies but constant gradient and drifts, into a bunch train with a

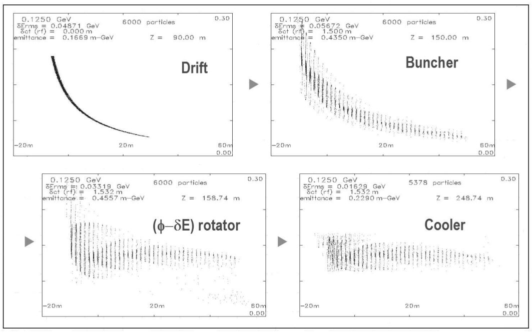

Fig. 3. Beam dynamics in the buncher and phase rotator

longer time duration and a lower energy spread. Example 2D simulation of the dynamics of the particles in the structure is shown on the Fig.3. The beam at the end of the buncher and phase rotation section has an average momentum of about 220 MeV/c. The proposed [6], [7] system is based on standard rf technology, and is expected to be much more cost effective than the induction-linac-based system considered in [5]. An additional benefit of the rf-based system is the ability to transport both signs of muon simultaneously.

To set up buncher parameters we choose some ideal particle to be the main central particle of the beam. Usually this is a particle with the coordinaets in the center of the beam particles coordinates distribution. We then set phases of rf cavities in such a way that this particle passes every cavity in the same phase ($\phi_s = 0$) of E field oscillations. By the virtue of the equations of motion in such a structure (see [8]), particles near the central one in ($\phi - \delta\mathrm{E}$) phase space are then formed into a stable group called "bunch", which oscillates around the central particle during its motion in the structure in this phase space. Because of the specific choice of the main central particle's phase and cavities parameters, we also have some other particles passing all cavities in the same $\phi_s = 0$ phase and, by the same equations of motion, bunches are formed around these particles as well. In following text we will call them "central particles" and the one chosen first "main central particle". Of course, all central particles are not real particles, they are just an idealization chosen to make equations of motion simpler.

Each cavity in the buncher has its frequency set to maintain the following condition: the time of arrival difference between two central particles in a place of rf field application remains equal to a fixed integer number of rf oscillations periods and this condition is maintaned as the beam propagates through the buncher

$$\Delta t = t_\mathrm{n} - t_\mathrm{c} = z\left(\frac{1}{v_\mathrm{n}} - \frac{1}{v_\mathrm{c}}\right) = nT_\mathrm{rf} = n\frac{\lambda_\mathrm{rf}}{c}, n \in \mathbb{Z}\,, \quad (1)$$

where n is the number of the bunch counted from the main central particles's one, t_c, t_n and v_c, v_n are time-of-arrival of main central and n-th central particle (main central particle has $n = 0$) and their velocities respectively, T_rf is the period of rf field oscillations, λ_rf is rf wavelength, c — speed of light.

As we set E field phase in rfs to be zero for the main central particle, for other central particles it is also zero so they pass the rfs when the field has zero strength and therefore their energies stay constant through the buncher. We keep the final frequency of the buncher and rotator fixed because of matching into 201.25 MHz cooling and/or accelerating sections, so setting in (1) $n = 1$, $\lambda_\mathrm{rf} = \bar{\lambda}$, $z = \bar{L}$, where z is the longitudinal coordinate with $z = 0$ at the beginning of the drift, $\bar{\lambda}$ is the final rf wavelength in buncher (defined by matching to the following cooler), $\bar{L}$ is the longitudinal coordinate of the last rf in buncher, we can define

$$\delta\left(\frac{1}{\beta}\right) = \left(\frac{1}{\beta_1} - \frac{1}{\beta_\mathrm{c}}\right) = \frac{\bar{\lambda}}{\bar{L}}\,, \quad (2)$$

where β_c, β_n are normalized main central particle and n-th central particle's velocities, and then rewriting (1) we get

$$\frac{1}{\beta_\mathrm{n}} = \frac{1}{\beta_\mathrm{c}} + n\delta\left(\frac{1}{\beta}\right)\,. \quad (3)$$

Therefore for kinetic energies of central particles in the buncher we have the following relation

$$T_n\left(\beta_\mathrm{c}, \delta\left(\tfrac{1}{\beta_\mathrm{c}}\right)\right) = W_0 \cdot \left(\left(1 - \left(\frac{\beta_\mathrm{c}}{1 + n\beta_\mathrm{c}\delta\left(\frac{1}{\beta_\mathrm{c}}\right)}\right)^2\right)^{-1/2} - 1\right)\,, \quad (4)$$

where W_0 is the rest energy of the particle, T_n is the kinetic energy of the n-th central particle. From (2),(3) it follows that in order to keep the time of arrival difference between two central particles constant, the frequencies of rfs in a buncher should depend on the longitudinal coordinate through

$$\lambda_\mathrm{rf}(z) = z \cdot \delta\left(\frac{1}{\beta}\right) \Rightarrow \nu_\mathrm{rf}(z) = \frac{c}{z \cdot \delta\left(\frac{1}{\beta}\right)}, \quad (5)$$

In the buncher the rf gradient is adiabatically increased over the length of the buncher. The goal here is to perform an *adiabatic capture*, in which the beam within each bunch is compressed in phase so as to be concentrated near the central particle's phase. We arbitrarily choose gradient to be increasing quadratically

$$V_\mathrm{rf}(z) = B\frac{(z - z_\mathrm{D})}{L} + C\frac{(z - z_\mathrm{D})^2}{L}, \quad (6)$$

where V_rf is rf voltage, z_D is the longitudinal coordinate of the beginning of the buncher (equal to the drift length), B and C — positive constants, defined by chosen initial and final rf gradfients in a buncher, L is a length of the buncher. Note that, since each of the bunches is centered at different energy, they all have different longitudinal oscillation frequencies, and a simultaneously matched compression for all bunches is not possible. Instead a quasi-adiabatic capture resulting in an approximate bunch length minimization in each bunch is attempted.

Following the buncher is the so-called ($\phi - \delta\mathrm{E}$) *vernier rotation system* in which the rf frequency is almost fixed to the matched value at the end of the buncher and the rf voltage is constant. In this system the energies of the central particles of the low-energy bunches increase, while those of the high-energy bunches decrease. So the whole energy spread reduces to the point where beam is a string of similar-energy bunches, a could be captured into the ~200 MHz ionization cooling system matched to the central energy of the beam.

Let us describe the rotator parameters calculation in more detail. At the end of the buncher we choose two reference particles (n_1 and n_2) kept $(n_2 - n_1)$ rf periods from each other along the buncher and the *vernier offset* δ. We then keep the second central particle at $((n_2 - n_1) + \delta)\lambda_{\rm rf}$ wavelengths from the first one through the rotator. So now it passes all rf cavities in a constant accelerating phase $\phi_{\rm n_2}$ having constant energy change $\Delta T_{\rm n_2}$ and after $|T_{\rm n_1} - T_{\rm n_2}| / \Delta T_{\rm n_2}$ cavities, energies of the first particle (usually we choose main central particle as first central particle) and the chosen second central one will be nearly equal. From this consideration we can derive the relation between the energy change of the n-th central particle in each cavity of the rotator and the rotator parameters:

$$\Delta T_n(E_{\rm rf}, \delta, n_1, n_2) = E_{\rm rf} \sin\left(2\pi\delta \frac{n - n_1}{n_2 - n_1}\right) , \qquad (7)$$

where ΔT_n is the energy change of the n-th central particle, δ is the vernier parameter, n_1 and n_2 are the numbers of chosen central particles, $E_{\rm rf}$ is the rf gradient of the cavities in rotator. This process also aligns the energies of other central particles and their bunches, hence at the end of the rotator we have the beam rotated in $(\phi - \delta{\rm E})$ space with significantly reduced energy spread. Simulation of the process in $(\phi - \delta{\rm E})$ phase space is shown on Fig.3.

Combining equations (4) and (7) we get the relation for the central energy of the n-th bunch after buncher and phase rotator:

$$T_n^{\rm fin}\left(\beta_{\rm c}, \delta\left(\frac{1}{\beta_{\rm c}}\right), E_{\rm rf}, \delta, n_1, n_2\right) = T_n + m\Delta T_n \ , \qquad (8)$$

where m is the number of rf cavities in rotator.

IV. Possible Variations

Some considerations about possible variations of the concept could be made. First, the concept originally assumes the use of different rf cavities in buncher. If we, for example, need buncher with 60 cavities in it then we should make 60 cavities each with different gradient and frequency. Producing such an array could be quite expensive and to reduce the overall cost we might think of decreasing the parameters diversity. Instead of 60 different cavities each with unique parameters we might use 10 rfs with 6 cavities having equal params or even less. For this case we need to study the dependence of the the structure performance on the "discretization" level. We also might think of making shorter section and different final central energy to decrease overall cost and/or to better fit to the recently-proposed FFAG accelerating section. This also leads to smaller number of bunches in a beam and larger muon losses. After the bunching section

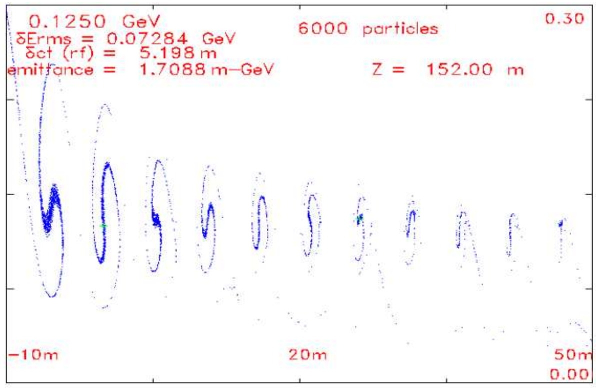

Fig. 4. Uses 60m drift + 90m 100→50 MHz rf (¡3MV/m) 180MV total

muons should be cooled and cooling section also employs rf resonators with certain frequency. Another constraint which should be fulfilled is matching the beam coming from the buncher into the cooler. As there are many different cooling section schematics proposed, there could be many different bunching sections best suited for them (see, for example Fig.4).

V. Problem Description and Key Parameters/Controls

The concept of the buncher and phase rotator is defined in the previous section, and it could be easily seen that there are many variations in the structure parameters leading to different operational characteristics and the total cost. The Neutrino Factory is now in R&D stage so there may be many different scenarios of the lattice usage and so many different optimizational constraints defined: minimal cost, different possible final rf frequencies, reduced number of rf frequencies and gradients in the structure, shorter/longer bunch trains, optimal number of muons captured, optimal desired central energy, fitting into allowed final energy spread etc. Matching into the accelerating/cooling structures following the buncher-rotator system and the transverse beam dynamics should also be considered. That is to say the problem of developing optimization scheme for the buncher and phase rotator with differrent imposed constraints has practical meaning and importance.

As could be seen from the structure description and relation given above, as the controls for the structure we can take following parameters:

1) *Drift*: the length of the section $L_{\rm D}$. Future studies, which include transverse motion, must also consider the apertures and focusing fields (this study uses fixed-field solenoids for transverse focusing). These focusing parameters are also critical for system performance.
2) *Buncher*: the length of the section $L_{\rm B}$, rf voltages $V_{\rm B}^i$, $i = \overline{1, n_{\rm rfs}}$ or initial and final voltage and the law of voltage increase (linear, quadratic, etc). Final frequency is usually strictly specified by the cooling/accelerating subsections of the whole accelerator, but could also be varied to find optimum.

3) $(\phi - \delta\mathrm{E})$ *Rotator*: the length $L_{\phi\mathrm{R}}$, rf voltage $V_{\phi\mathrm{R}}$ of the phase-energy rotation section, number N of rf field oscillation periods between chosen second central particle and the main central particle (with $n = 0$), and the vernier parameter δ. Also the kinetic energy T_c of the main central particle could be changed (usually we take T_c to be the peak of energy distribution of beam's particles).

VI. Optimization Approach and Realization

In this paper we propose an approach to optimize final energies spread and desired central beam energy at the end. This approach is based on the fact that motion of the particles in the buncher and phase rotator is well described by the equations of motion of the central particle of the bunch in laboratory coordinate system and equations of motion in coordinates relative to those of the central particle for other particles of the beam. This means that in reducing energy spread of the particles and bringing their energies close to desired one we should align energies of the central particles around this energy and then energies of all other particles of respective bunches will be automatically aligned around it. To measure how good buncher and phase roator with chosen set of controls perform in aligning central particles energies around desired energy we can use exact formula (8).

To do so we introduce the merit (or penalty) function which has the form of the sum of the squared distances from the final energy of the i-th central particle of the bunch T_i^{fin} calculated by (8) to desired final central energy of the beam $\bar{T}$:

$$I = \sum_{i=b_1}^{b_2} c_i (T_i^{\mathrm{fin}} - \bar{T})^2 \,, \tag{9}$$

where b_1 and b_2 are is the first and last bunches of interest, c_i are the weight coefficients which allow us to put more emphasize on some bunches (for example with more particles in them). In our optimization runs we use two penalty functions of the form (9). In the first function we set

$$c_i = 1 \,,\ i = \overline{b_1, b_2}$$

i.e. we assume all bunches to be equally important to us and we impose equal penalties if two central particles of different bunches are on the same distance (in energy) from the $\bar{T}$:

$$I_1 = \sum_{i=b_1}^{b_2} (T_i^{\mathrm{fin}} - \bar{T})^2 \,. \tag{10}$$

In the second penalty function we use the beam of 5000 particles coming from the target (generated by MARS code [9]) to calculate weight coefficients

$$p_i = \frac{n_i}{N} \,,\ i = \overline{b_1, b_2}$$

where n_i is the number of particles in i-th bunch, N is the total number of particles in the beam, i.e. if two central particles of different bunches are on the same distance from the $\bar{T}$ then the penalty is larger for the bunch with more particles in it:

$$I_2 = \sum_{i=b_1}^{b_2} p_i (T_i^{\mathrm{fin}} - \bar{T})^2 \,. \tag{11}$$

Both these functions depend on the buncher and phase rotator design parameters ($\bar{T}$, $\bar{L}$, $\bar{\lambda}$, n_1, n_2, δ, E_{rf}, m), so by changing these parameters (or part of them) to minimize penalty functions we obtain lattice doing better in aligning particles around desired central energy and so better reduce the overall energy spread.

Optimization routine was implemented and tested in *COSY Infinity* [10]. Built-in simplex and simulated annealing optimizers [11] were used to find a good fit for objective function. Example of the optimization run with penalty functions (10), (11) is shown on the Fig.7. Initial parameters are taken from the demonstration of the principles of the buncher and phase rotator article [7]. More interesting is the optimization of the last Feasibility Study – Study 2a parameters of the lattice. The curves before optimization and after optimization with penalty function (11) are shown on the Fig.5. Number of the particles in bunches used to calculate weight coefficients is shown on the Fig.6. Clearly

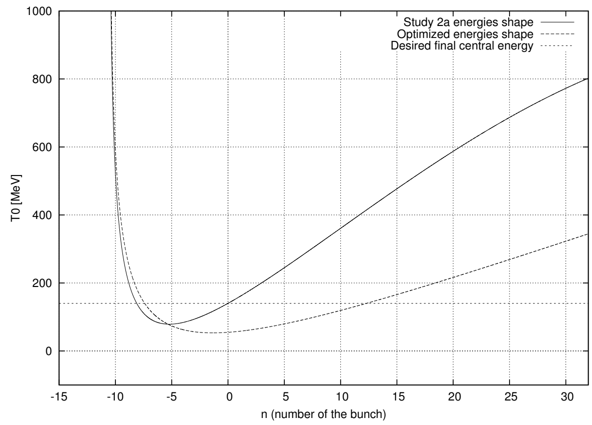

Fig. 5. Optimization of the central energies shape, penalty function 2 (Study 2a parameters)

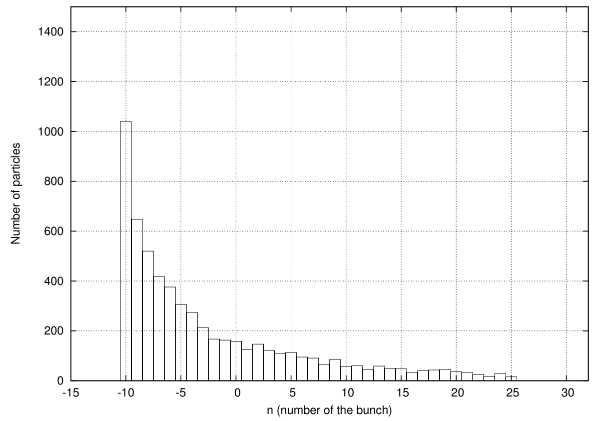

Fig. 6. Number of particles in bunches

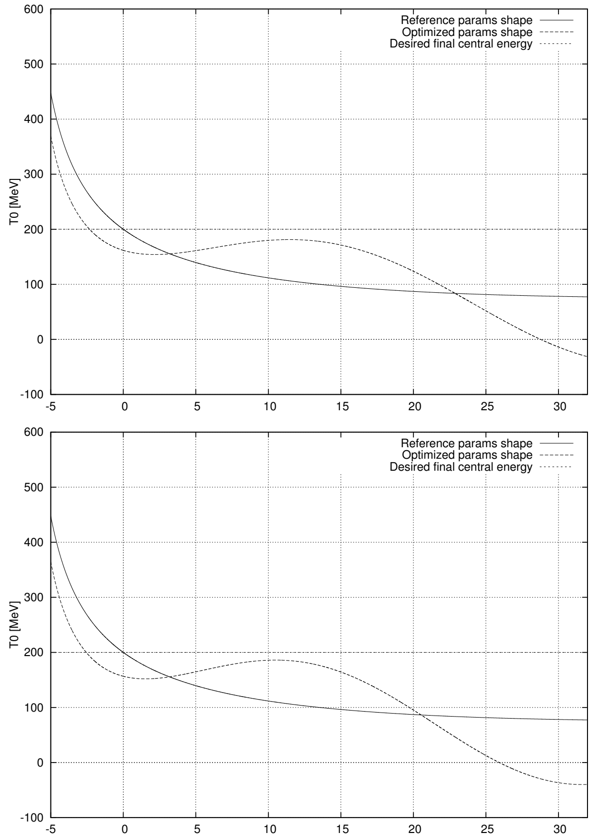

Fig. 7. Optimization of the central energies shape, penalty functions 1 (left) and 2 (right) (example)

optimized set of parameters is performing better in aligning the central particles energies (the penalty function value is reduced from 485808.12 to 316581.03 i.e. more than for 30%).

VII. Summary and Future Plans

In this article an approach for optimization bunch central energies shapes (ans so the overall energy spread) in the buncher and phase rotator is proposed and described. The described approach is implemented in *COSY Infinity* code. The code is tesed on different examples and subsets of optimization parameters, optimization of the last Study2a parameters is presented.

After the natural decomposition of the problem of optimizing particles dynamics in buncher and phase rotator and solving first simplified problem we need to check if optimized parameters do better for the whole beam. For this purpose we need to perform complete 6D simulations of the beam dynamics in some simulation code (*COSY Infinity*) and in de-facto standard simulation code for the the Neutrino Factory and Muon Collider Collaboration *ICOOL* [12]. We also want to build more sophisticated merit functions to inlcude particles capture in buckets, cost of the structure, etc. to perform optimization on different structure parameters. And in the end we want to have a tool which would allow us to easily obtain optimal sets of parameters of buncher and phase rotator for different designs of target, cooling and accelerating channel sections, because the project is still in the active development stage, new ideas are coming up and there is no completely stable design available.

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