

Isochronous (CW) Non-Scaling FFAGs: Design and Simulation

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Abstract. The drive for higher beam power, high duty cycle, and reliable beams at reasonable cost has focused international attention and design effort on fixed field accelerators, notably Fixed-Field Alternating Gradient accelerators (FFAGs). High-intensity GeV proton drivers encounter duty cycle and space-charge limits in the synchrotron and machine size concerns in the weaker-focusing cyclotrons. A 10-20 MW proton driver is challenging, if even technically feasible, with conventional accelerators – with the possible exception of a SRF linac, which has a large associated cost and footprint. Recently, the concept of isochronous orbits has been explored and developed for nonscaling FFAGs using powerful new methodologies in FFAG accelerator design and simulation. The property of isochronous orbits enables the simplicity of fixed RF and, by tailoring a nonlinear radial field profile, the FFAG can remain isochronous beyond the energy reach of cyclotrons, well into the relativistic regime. With isochronous orbits, the machine proposed here has the high average current advantage and duty cycle of the cyclotron in combination with the strong focusing, smaller losses, and energy variability that are more typical of the synchrotron. This paper reports on these new advances in FFAG accelerator technology and presents advanced modeling tools for fixed-field accelerators unique to the code COSY INFINITY.

Keywords: FFAG design, lattice, simulation, isochronous, CW, proton driver

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INTRODUCTION

Accelerators are playing increasingly important roles in science, technology, and medicine including accelerator-driven subcritical reactors, industrial irradiation, material science, neutrino production, and cancer therapy. The drive for higher beam power, high duty cycle, and reliable beams at reasonable cost has focused world attention on fixed field accelerators, notably Fixed-field Alternating Gradient accelerators (FFAGs) particularly in reference to advanced proton drivers [1]. High-intensity GeV proton drivers encounter duty cycle and space-charge limits in the synchrotron and machine size concerns in the weaker-focusing cyclotrons. For example, a 10-20 MW proton driver is challenging, if even technically feasible, with conventional accelerators – with the possible exception of a SRF linac, which has a large associated cost and footprint.

Recently, the concept of isochronous orbits has been explored and developed for nonscaling FFAGs using powerful new methodologies in FFAG accelerator design and simulation. The property of isochronous orbits enables the simplicity of fixed RF and, by tailoring a nonlinear radial field profile, the FFAG can remain isochronous beyond the energy reach of cyclotrons, well into the relativistic regime. With isochronous orbits, the machine proposed here has the high average current advantage and duty cycle of the cyclotron in combination with the strong focusing, smaller losses, and energy variability that are more typical of the synchrotron. With the cyclotron as the current industrial and medical standard, a competing CW FFAG would have broad impact on facilities using medical accelerators, proton drivers for neutron production, accelerator-driven nuclear reactors, waste transmutation, and the production of radiopharmaceuticals, and open up a range of as-yet unexplored industrial applications. Further, a high-intensity proton driver is a critical technology for the Neutrino Factory and Muon Collider. This paper reports on these new advances in FFAG accelerator technology, design, and simulation,

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presenting advanced tools recently developed for all variations in fixed-field accelerators and unique to the code COSY INFINITY.

FFAGS: BACKGROUND AND CLASSIFICATIONS

The FFAG concept in acceleration was invented in the 1950s independently in Japan [2], Russia [3] and the U.S. [4]. The field is weak at the inner radius and strong at the outer radius, thus accommodating all orbits from injection to final energy. Focusing is provided by an alternating gradient. An extensive discussion of the various FFAG configurations, including derivations of the formulas relating the various accelerator and orbit parameters, can be found in the references [5]. The configuration initially proposed was called a radial sector FFAG accelerator. A spiral sector configuration was also invented consisting of magnets twisted in a spiral such that, as the radius increases and the beam crosses the magnet edges, it experiences alternating gradients. With no reverse-bending magnets, the orbit circumference of the spiral-sector scaling FFAG is about twice that for a circular orbit in a uniform field. These machines are the so-called scaling FFAGs (either spiral or radial-sector FFAGs) and are characterized by geometrically similar orbits of increasing radius. Direct application of high-order magnetic fields and edge focusing maintains a constant tune and optical functions during the acceleration cycle and avoids low-order resonances. The magnetic field follows the law $B \propto r^k$, with r as the radius and k as the constant field index.

The non-scaling FFAG was invented in 1997 (C. Johnstone and F. Mills) and a working lattice published in 1999 [6] as a solution for the rapid acceleration of muon beams. The non-scaling FFAG proposed for muon acceleration utilizes simple, combined function magnets like a synchrotron. However, it does not maintain a constant tune and is not suitable for an accelerator with a modest RF system and therefore a slower acceleration cycle.

Recently, innovative solutions were discovered (C. Johnstone, Particle Accelerator Corp.) for non-scaling FFAGs which duplicated the constant tune feature of the scaling FFAG without applying the scaling principle. This new non-scaling FFAG accelerator applied weak and alternating gradient focusing principles (both edge and field-gradient focusing) in a specific configuration to a fixed-field combined-function magnet to stabilize tunes [7]. Note that stable tunes, however, do not imply isochronous orbits (a constant revolution frequency of the beam).

Isochronous orbits are achievable only at relativistic energies in a synchrotron and predominately non-relativistic energies in a cyclotron. In a synchrotron, the magnetic field increases proportional to energy and therefore particles are confined about a laboratory-based reference trajectory independent of energy. Since the path-length is fixed independent of energy, a frequency change is required except at highly-relativistic energies, so swept-frequency RF is unavoidable. In a fixed-field machine, such as a FFAG or cyclotron, the reference orbit moves outward transversely with energy. The orbital path length and, in general, the orbital frequency changes with energy (with only a constant magnetic confinement field, the energy reach of an isochronous cyclotron is limited).

As noted above, even more recently, the problem of isochronous orbits has been solved for non-scaling FFAG designs in the energy regime of interest – ~ 1 GeV, but also lower energy. These isochronous, compact non-scaling FFAGs lattices were discovered by tailoring an arbitrary radial field profile to confine orbits to isochronous ones and simultaneously constraining machine tunes using new advanced methodologies in accelerator design and modeling. Designing and demonstrating performance, particularly for the FFAGs with their complex field profiles and edge contours, required new advances in accelerator tools that will be described in the following section.

DESIGN AND SIMULATION TOOLS

A major prerequisite for advanced accelerator design is the existence of reliable, easy to use optimization and simulation tools. Such tools are different in nature for FFAGs than those used in other kinds of accelerators; the rapidly azimuthally varying fields entail significant fringe field effects and out-of-plane nonlinearities. Tracking of orbits for assessment of dynamic aperture needs to be carried out with careful consideration of the nonlinearities, and utilizing modern methods of symplectification to compensate spurious effects due to nonphysical violation of phase space volume conservation. Further, space charge effects inherent in high-power operation of the devices produce very novel challenges due to the necessity to treat crosstalk with many neighboring orbit. Optimization challenges are difficult since they always affect many orbits at the same time and hence need to be of a global nature.

The ability to model FFAGs – both scaling and non-scaling – with conventional codes is limited. Often new prototypes of accelerators including FFAGs are simulated with codes like MAD [8] and Optim [9] as the standard

codes for modeling, but these codes do not provide much flexibility in the description of the available fields and are limited to low order. This limitation can be inadequate to fully demonstrate performance including dynamic aperture, where strong nonlinearities due to edge fields and other high-order effects appear. The significant size of the beam emittance nominally invalidates the paraxial representation (kinematical, or angle effects in the Hamiltonian are significant), which implies that codes that fully represent the kinematics are necessary.

The cyclotron code CYCLOPS [10] has been used to describe the FFAG, but has limited accuracy in this application primarily due to lack of out-of-plane expansion order, which specifically impacts the ability to describe dynamic aperture especially in the case of edge effects with rapid field fall-off – a condition that appears in the FFAG but is not normally present in cyclotrons. Field expansion codes such as ZGOUBI [11] can accurately track the kinematics of such machines, but they have limitations when field profiles become very complex and include significant nonlinear effects. Further, ZGOUBI [11] requires dedicated effort and expertise in order to implement a FFAG design accurately, cannot easily deal with the large transverse emittances required, and lacks some modern analysis tools for symplectic tracking, global optimization, tunes shifts and chromaticities, and resonance analysis. In particular, field map codes are difficult to use when one wants to study parameter dependencies, perform detailed study of dynamic aperture, extract advanced optical functions such as high-order resonances, or use optimization routines to study the most advantageous combination of multipole correction schemes, for example.

Modern extensions of the transfer map-based philosophy [12], as implemented in the arbitrary order code COSY INFINITY [13], can remedy the limitation in order and in the accuracy of the dynamics. It is particularly suitable for accurate, high-order descriptions of accelerators. Yet in their standard configuration based on pre-selected field elements like combined function magnets with edge angles, they are still not sufficient to describe in full detail the richness of the nonlinearities that can arise in the fields.

Significant enhancements of the code COSY INFINITY for the particularly challenging case of FFAG accelerators have been implemented. Based on the Differential Algebra (DA) approach [12], unconventional arbitrary-field elements comprising the machine can now be described in a conventional matrix formulation to any order, without any approximations in the dynamics. The following is a list of the powerful features developed for accelerators and native to COSY INFINITY.

- **Arbitrary Order Maps** COSY allows the computation of all dynamics of the system to arbitrary order, including out-of-plane expansions of fields and any nonlinear terms in the Hamiltonian.
- **Arbitrary Fields** There is no principal limitation of the fields COSY can treat, as long as they can be modeled in a reasonable way. For efficient initial simulation and optimization, it is particularly useful to utilize very high order out-of-plane expansions of suitable midplane models.
- **Symplectic Tracking** There are various methods to perform tracking in COSY that preserve the symplectic symmetry inherent in Hamiltonian systems, including methods that do so with minimal modifications based on the Expo approach [14]. These allow a very faithful estimation of dynamic aperture.
- **Nonlinear Analysis Tools** In addition to the mere empirical study, there are various tools for analysis of nonlinear effects, including the normal form-based computation of high-order amplitude-dependent tune shifts and resonances.
- **Sophisticated Global Optimization** COSY allows the automatic adjustment and optimization of arbitrary system parameters; and different from other tools, the search uses methods of global optimization with constraints over a pre-specified search region, and not merely local optimization from a starting parameter setting.

In practice, different accelerators are described accurately by different orders in the matrix, or map. For design studies, often orders around 5 or 7 are sufficient; however, once a specific or optimal configuration is chosen, final tracking studies are usually pursued at the 11th – 15th order for required accuracy in predicting performance.

Examples: 6-fold Symmetric, 4 MeV FFAG vs. Cyclotron

To provide an illustrative example, a sample FFAG having six-fold symmetry was studied, with focusing supplied by an azimuthal field variation expressed as a single Fourier mode, as well as edge focusing. The system is studied to various orders of out-of-plane expansion, so that conclusions about dynamic aperture can be drawn. The results for orders three and five, which are typical for the situation of conventional out-of-plane expansion in codes like CYCLOPS, are shown. Since the method used in COSY is not based on divided differences, the necessary in-plane derivatives can actually be calculated to any order desired with an accuracy that is always close to machine precision [12].

The results of tracking without symplectification and with Expo symplectification are shown in Figures 1-3. The Expo symplectification scheme is known to minimize the alterations to the non-symplectic tracking results compared to other symplectification methods. Still, symplectification greatly affects the inferred dynamic aperture of the system.

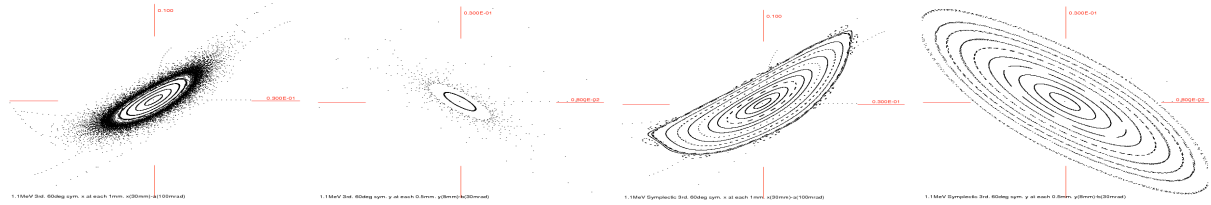


FIGURE 1. Tracking in a model non-scaling FFAG with third-order out of plane expansion, without symplectification (left) and with symplectification (right).

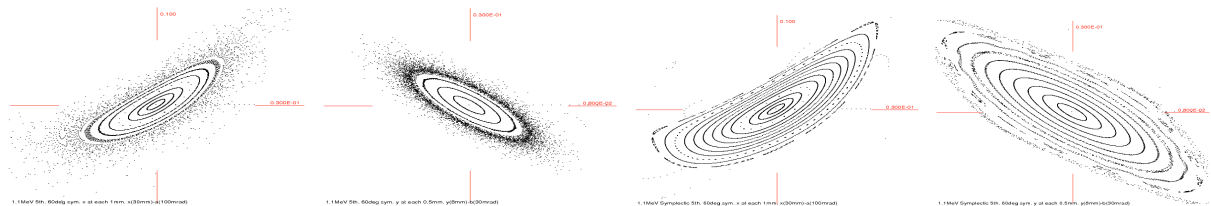


FIGURE 2. Tracking in a model non-scaling FFAG with fifth-order out of plane expansion, without symplectification (left) and with symplectification (right).

However, Figure 3, which is based on order eleven out-of-plane expansion, show significant additional effects and different dynamic aperture, suggesting that the low order methods for out-of-plane expansion and dynamics are not sufficient to capture the details of the dynamics. It would in fact lead to an incorrect prediction of dynamic aperture, underestimating it in the horizontal direction and overestimating it in the vertical. Further increases in order beyond eleven do not significantly affect the details of the symplectic motion shown, but continue to influence the non-symplectic motion. A rough estimate reveals that in this particular case, the dynamics as seen in non-symplectic tracking seems to begin to stabilize around order 17, which is still rather easily obtained within the power of a modern workstation.



FIGURE 3. Tracking in a model non-scaling FFAG with eleventh-order out of plane expansion, without symplectification (left) and with symplectification (right).

FFAG and Cyclotron Design

A powerful new methodology has been pioneered for all fixed-field accelerator optics design (FFAGs and cyclotrons), using control theory and optimizers to develop executable design scripts. These procedures allowed global exploration of all important machine parameters in a simplified lattice. With this methodology, the stable machine tune for FFAGs, for example, was expanded over an acceleration range of 3 up to 6 in momentum with linear fields and up to 44 with nonlinear fields and included optimization of complex edge contours, footprint, and components. Full evaluation of the starting lattice, however, required new advanced simulation tools not existing in current accelerator codes. Such tools have been developed and implemented as an add-on to COSY INFINITY; the

FACT (FFAG And Cyclotron Tools) accurately predict and optimize machine performance (Figure 4). A 3D field expansion in polar coordinates is one output format which can be used by other codes (Figure 4). The starting points from the design scripts are directly imported into and modeled in COSY INFINITY using FACT software.

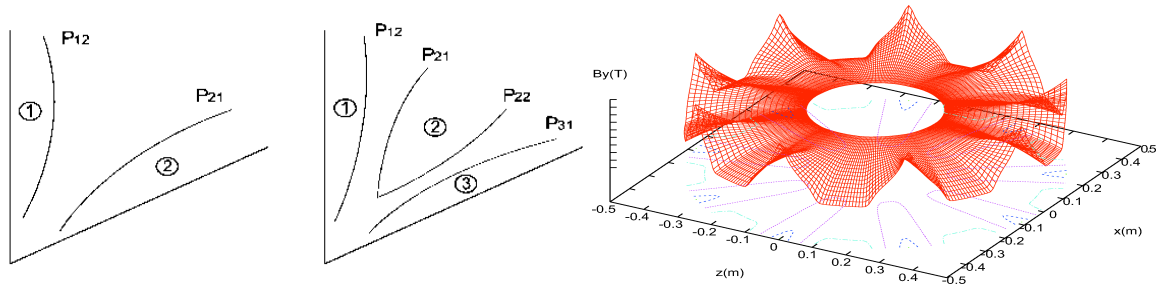


FIGURE 4. Complex edge profiles supported in COSY (left) and realistic 3D field expansion output (right) by COSY in polar coordinates derived from simple starting design lattices.

Using this powerful methodology, scripts were implemented to design and optimize a FFAG and provide results for an equivalent cyclotron (both low-energy, 4 MeV designs) shown in Figure 5. The isochronous sector cyclotron employs a 5 kG field. The FFAG initiates injection at 5 kG, but the field rises to 1 T at extraction which allows longer straight sections between magnets and improves extraction efficiency.

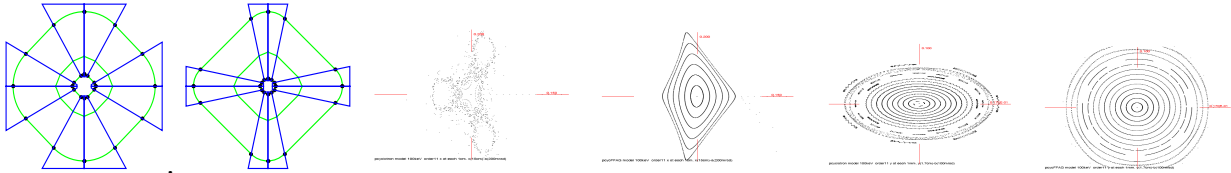


FIGURE 5. Subtleties in the transverse dynamics of a cyclotron and an equivalent FFAG at 100 keV: horizontal cyclotron and FFAG (left pair) and vertical cyclotron and FFAG (right pair) as observed in advanced tracking simulations in COSY.

ISOCHRONOUS FFAG DESIGN

The concept of isochronous orbits has been tested on a preliminary 0.25-1 GeV non-scaling FFAG designed using the new methodologies and optimizers described above. The ring layout, 3D field profile, and tracking results are given in Figure 6 and Table I lists general parameters. Figure 7 shows corresponding results achieved by Craddock, et al. [15] using the cyclotron code, CYCLOPS showing isochronous behaviour is $\pm 3\%$ in this preliminary design.

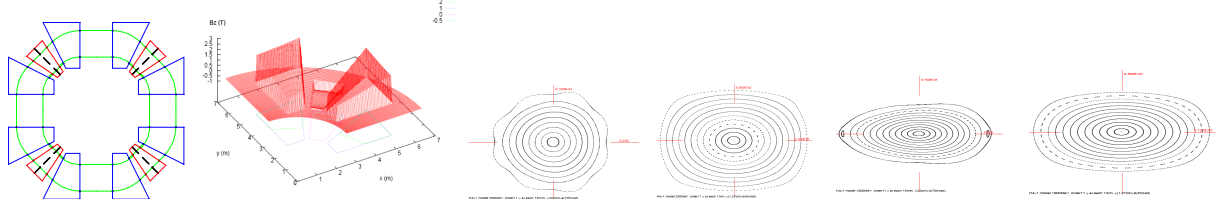


FIGURE 6. Ring layout and 3D field profile from COSY (left two). Tracking profiles at injection (left pair) and extraction (right pair) in horizontal (1.5 mm steps) and vertical (1mm steps), respectively.

TABLE 1. General Parameters of an initial 0.250 – 1 GeV non-scaling, isochronous FFAG lattice design.

Parameter	250 MeV	585 MeV	1000 MeV
Avg. Radius (m)	3.419	4.307	5.030
Cell v_x / v_y (2π rad)	0.380 / 0.237	0.400 / 0.149	0.383 / 0.242
Ring v_x / v_y (2π rad)	1.520 / 0.948	1.600 / 0.596	1.532 / 0.968
Field F/D (T)	1.62 / -0.14	2.06 / -0.31	2.35 / -0.42
Magnet Length F/D (m)	1.17 / 0.38	1.59 / 0.79	1.94 / 1.14

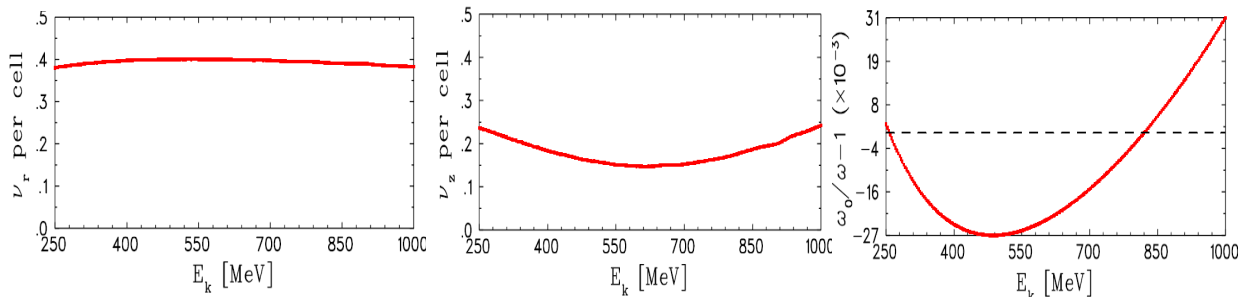


FIGURE 7. Cell tune radial (horizontal) and vertical using the cyclotron code, CYCLOPS and deviation from isochronous orbits (far right) showing $\pm 3\%$ variation in revolution frequency.

SUMMARY

Powerful new advanced accelerator design scripts have been developed using control theory and optimizers the results of which are directly imported into the advanced accelerator code COSY INFINITY. Various methods of describing complex fields and components are now supported in COSY and include representation in radius-dependent Fourier modes, complex magnet edge contours, as well as the capability to interject calculated or measured field data from a magnet design code or actual components. With these new tools, a high-energy isochronous FFAG has been designed and performance verified.

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