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Muon beam ionization cooling in a linear quadrupole channel

C. Johnstone[a,*], M. Berz[b], D. Errede[c], K. Makino[c]

[a] Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA
[b] Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA
[c] Department of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green Street, Urbana, IL 61801-3080, USA

Abstract

In a scenario for either a Neutrino Factory or Muon Collider, the anticipated transverse beam emittance subsequent to capture and phase rotation is so large that it permits a relaxation of the requirements on beam spot size in the early stages relative to the final stages of ionization cooling. Staging the cooling process according to initial emittances, coupled with modest cooling factors, permits more optimal and efficient cooling channel designs and avoids much of the difficulty encountered with channels which attempt to maintain strong focusing (large, 300–500 mrad, divergences) across ultra-large momentum ranges ($\geqslant \pm 20\%$ $\delta p/p$). Relaxation of spot size at the absorber, especially in the "precooling" stage, allowed development of an efficient transverse cooling channel based simply on a quadrupole FODO cell. This work describes the design of such a cooling channel and its application as an upstream stage of beam cooling. Being a linear channel with no bends, it serves to reduce the large transverse beam size delivered from muon-beam capture and bunching before entering more restricted optical structures such as transverse plus longitudinal cooling channels or accelerators.

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Keywords: Beam optics; Neutrino factory; Muon cooling channel

1. Introduction

The production, acceleration, and storage of a muon beam sufficiently intense to drive a Neutrino Factory or Muon Collider requires multi-stage preparation. A diagram of the important stages in the path to a Muon Collider [1] (left) and a Neutrino Factory [2] (right) is given in Fig. 1. Muons are created through the decay of pions, which are produced by directing an intense beam of protons (10^{14}/pulse) onto a production target. A proton driver with an ultra-short beam pulse (3 ns long) is considered the initial stage of a Neutrino Factory (or Muon Collider). Collection, capture and bunching of large transverse and longitudinal emittance pion/muon beams immediate to and downstream of the production target form the basis for the next major systems. A "cooling stage" for emittance reduction follows, but precedes acceleration and is the subject of this work. The acceleration and storage rings, albeit nonconventional due to the large admittance and rapid cycle requirements that are imposed by still large emittances and short muon lifetimes, respectively, represent the final stages of these facilities.

*Corresponding author.
E-mail addresses: cjj@fnal.gov (C. Johnstone), berz@msu.edu (M. Berz), derrede@uiuc.edu (D. Errede), makino@uiuc.edu (K. Makino).

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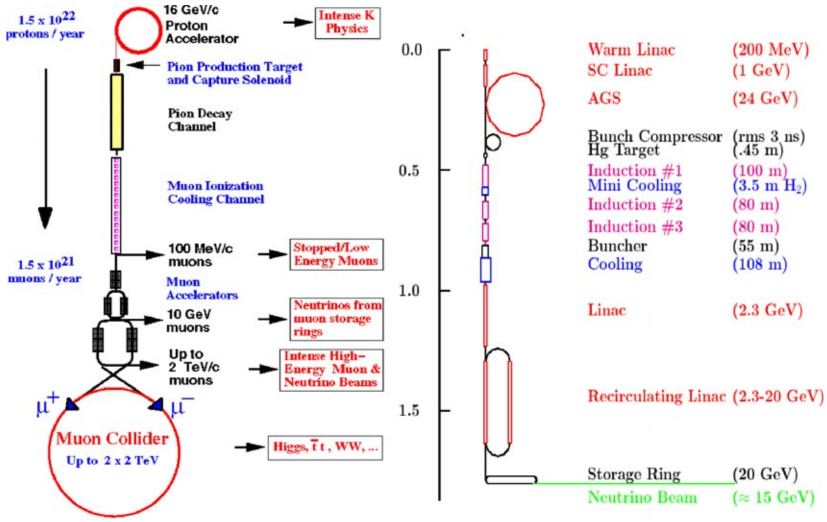


Fig. 1. Schematics of a Muon Collider [1] (left) and a Neutrino Factory [2] (right).

All scenarios for either a Neutrino Factory or a Muon Collider require acceptance and capture of unprecedented emittances both longitudinally and transversely. Acceleration of intense muon beams is not practical without a significant reduction, or cooling, of incipient beam emittances—by at least a factor of 10 for a Neutrino Factory [2,3] and at least a factor of 1000 for a Muon Collider [1]. Achieving the required emittances is possible because muons are heavy leptons and do not interact strongly in material, with energy loss occurring through ionization. Emittance reduction occurs because the muon beam loses momentum in all directions when traversing a target, or so-called absorber, and this energy loss can be replaced solely in the longitudinal direction by re-acceleration in an RF cavity; thus decreasing the beam's divergence for a given transverse dimension. The technique of ionization cooling permits reduction of transverse emittances (4D phase space), or beam sizes to levels acceptable for injection into large-acceptance accelerators, or ring coolers [4]. Ring coolers are multi-turn cooling channels designed to further reduce the transverse plus longitudinal emittance (6D phase space) to the smaller values required by certain acceleration scenarios or for a Muon Collider. However, ring cooling requires a "precooled" beam as the design cannot accept the ultra-large emittances found upstream and, therefore, represents an enhancement, or advanced stage of cooling beyond the baseline described here. A simplistic view of cooling for a Neutrino Factory is illustrated in Fig. 2.

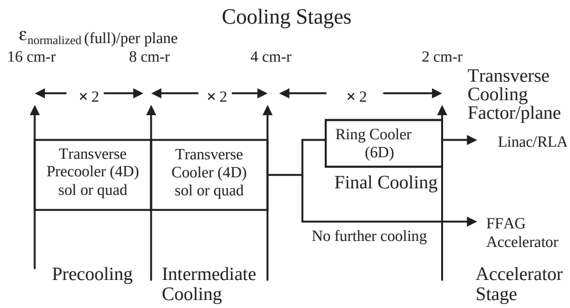


Fig. 2. Rough schematic of the stages in cooling for a Neutrino Factory.

The designs of stable optical configurations for cooling channels are particularly challenging because the straightforward cooling dynamics described above compete with the stochastic processes in the absorber; predominately multiple, or Coulomb, scattering which re-heat the beam. Energy straggling is also important, but represents more of a momentum or longitudinal emittance effect. The quadrupole channel, as designed in this work, is relatively insensitive to energy fluctuations, but in the solenoidal channel there is significant re-heating from straggling mainly due to pathlength differences (the larger the energy or angle, the larger the helix about the guiding orbit in a solenoid) and there are significant transverse–longitudinal correlations which develop in the presence of the absorber. (Although the beam is assumed injected on the solenoid's central axis, this is approximate due to the large production angles and energy spreads collected in this system which make it difficult to identify a specific reference orbit for the beam distribution.) It is these stochastic processes combined with the ultra-large transverse and longitudinal beam emittances that dictate the optical design of any cooling channel.

At this point, a discrete staging approach to cooling has not been systematically addressed. In published studies [1–3], an adiabatic approach was adopted with ever-decreasing low-beta points, cell lengths, and corresponding physical apertures. A staged design, rather than a continuously changing one (even approximately), may realize a technical and cost advantage by potentially decreasing the number of components and considering non-superconducting elements where feasible.

This work describes a quadrupole-based stage designed to cool initial (full) emittances of about 8–10 cm (normalized) by a factor of two in each transverse plane. Its application is clearly as an upstream stage of cooling and, being a linear channel with no bends, serves to reduce the large transverse beam size in preparation for acceleration or for injection into ring coolers. As will be shown, this channel cools efficiently and beyond the momentum reach of a solenoidal-based precooler. It uses large bore, normal-conducting quadrupoles and in the simulations all higher-order effects including fringe fields are fully modeled.

2. Optics criteria for a Muon cooling channel

Clearly, a net cooling effect can be achieved only if the cooling terms surpass the reheating ones, a state achieved through proper optics design in a cooling channel. To design effectively, it is important to first understand re-heating due to multiple scattering because its dynamics ultimately limit the minimum transverse emittance that can be achieved in a particular cooling channel. The equation which follows represents the emittance increase in an absorber due to multiple scattering and, in the presence of cooling (reacceleration by an RF cavity), the minimum emittance achievable for a specific channel design. This minimum, the so-called equilibrium emittance, is characteristic of the channel's optics, occurring when the cooling term is equal to the re-heating term, and is determined almost exclusively by an analysis of multiple scattering [1]:

$$e_{\mathrm{N,min}} = \frac{\beta_{\perp}(14\,\mathrm{MeV})^2}{(2\beta m_{\mu} L_{\mathrm{R}} \cdot \mathrm{d}E/\mathrm{d}s)} \tag{1}$$

where $\beta_{\perp}$ is the transverse beta function at the absorber, β the relativistic factor, m_{μ} the mass of the muon, L_{R} the radiation length of the absorber material, and $\mathrm{d}E/\mathrm{d}s$ the energy lost per meter in the absorber.

Although the absorbers under consideration [5] are necessarily extended longitudinally in space and cover tens of centimeters, it is still accurate to consider the average beta function across the absorber as representative of the total increase in emittance, particularly for hydrogen where multiple scattering is minimal. (It is interesting to note that the average rms multiple scattering angle even for an extended target is just $\sqrt{n} \times \theta_{\mathrm{s}}$, where n is the average number of scatterings and θ_{s} is the rms scattering angle. So, in effect, the above equation is relevant for both point and extended targets as long as the beam dimension does not significantly change over the target due to multiple scattering.)

2.1. Linear optics

From an optics standpoint, it is clear from Eq. (1) that the lower the average beta across the target, the proportionally lower is the emittance increase from re-heating (multiple scattering), and therefore, the lower the achievable equilibrium emittance. This is easily understood by commenting that minimal beta functions produce maximal beam divergences. Since, statistically, the initial beam divergence (rms) and multiple scattering angle (rms) add in quadrature to characterize the outgoing beam, its emittance is minimized when the net contribution of the multiple scattering angle to the total beam divergence is minimized—which occurs at large beam divergences (or minimum beta) relative to multiple scattering. It is also interesting to note that the increase in beam emittance from multiple scattering is independent of the initial emittance. Therefore, the performance of a cooling channel depends only on the beta functions at the absorber and on their constancy across a large momentum range. To be effective, a cooling channel must be able to accept at least $\pm 20\%$ $\delta p/p$. So, in addition to transverse stability, momentum stability of the "low-beta" at the absorber implies that the total phase advance must be controlled, or varied slowly over a tremendous range in momentum.

2.2. Nonlinearities

Given the large transverse extent of the beam, clearly the most important nonlinearity is to be found in the fringe fields associated with the quadrupole. (For the solenoid, which is a soft-focusing element, the low-beta point at the absorber is achieved solely through fringe fields, with high-order nonlinearities that increase dramatically with aperture [6].) All simulations of the quadrupole channel incorporated full fringe fields and included the impact of different types of fringe fields on the performance.

3. Cooling channel design

The motivation for solenoidal focusing is clear from the equation for the equilibrium emittance. From its dual focusing action, identical optical properties can be achieved simultaneously in the two transverse planes with solenoids, allowing the absorber to be optimally positioned at the lowest-beta point both horizontally and vertically. As mentioned, the solenoid is a soft-focusing element with strong high-order effects that increase dramatically with aperture. This consideration, combined with the high-field requirements of solenoidal channels due to their weak fringe-field focusing, makes it attractive to consider simple linear focusing elements, or quadrupoles, in constructing cooling channels. Since the quadrupole can only focus in one plane, either a complex lens system (consisting of 2 or 3 alternating focusing/defocusing quadrupoles) is required to form a low-beta point in both planes, or the requirements on the beta functions at the absorber must be reviewed and relaxed in order to use a simpler alternating lens system (such as a FODO-based cooling channel). Optimization of cooling necessarily implies an evaluation of the optical function requirements for different stages of cooling.

4. Design and staging of cooling channels

The reference designs for cooling channels were driven by the transverse acceptances of the acceleration baseline, in particular by the recirculating linacs. The full acceptance achieved so far in the recirculating linacs lies between 1.5π and 2π cm-rad. Assuming this acceptance corresponds to 2.5σ of a Gaussian beam profile, the rms emittance demanded from cooling is 2.4 to 3.2π mm rad. Practically, channels do not cool efficiently near their equilibrium emittance. The sFOFO solenoidal cooling channels, for example, were designed for an equilibrium emittance of about 1.7π mm rad to insure sufficient emittance reduction for successful acceleration. To achieve such a low equilibrium emittance, assuming 35 cm of liquid hydrogen as the absorber material, the beta functions at the absorber are only 40 cm—values that are comparable to current collider interaction points and which conventionally require a final-focus telescope when implemented with quadrupoles.

For comparison, the full initial normalized beam emittance, as defined by the present bunching scheme, is about 16π cm-rad, or the expected order of magnitude differential quoted in the introduction. In terms of a Gaussian with a 2.5σ acceptance, σ would correspond to 2.6π cm-rad, implying a factor of 8–10 cooling for acceleration. Imposing the stringent equilibrium emittance requirement needed at the exit of cooling onto the initial stages complicates the upstream channel design and may ultimately compromise or strongly limit overall cooling performance. Clearly a very low-beta, or tight spot size, imposed on this ultra-large transverse emittance and over an unprecedented, $\pm 20\%$ $\delta p/p$, momentum range has severe implications in the design of a compact cooling channel.

In a study of sFOFO and quadrupole cooling performances through extensive and comprehensive simulations, it was determined empirically that the cooling rate of the beam was quite rapid up to about 50% above the equilibrium emittance. The number of channels required at this point to cool to the equilibrium emittance was at least a factor of 2 more than the number required to cool initially by a factor of 2. Further, the initial cooling rate is not strongly enhanced by lowering the equilibrium emittance. (Roughly, to increase the cooling rate significantly requires the equilibrium emittance to be decreased by an additional factor of 2, and this enhanced rate is only effective until $3\times$ the equilibrium emittance. A factor of 2 in emittance corresponds to a $\sqrt{2}$ increase in beam divergence, which implies an equivalent increase in aperture and associated image or spot-size aberrations.) This observation indicates that the extremely low-beta functions required in the latter stages of cooling are not a prerequisite during the early stages and may actually be less optimal from both a technical and nonlinear standpoint. If a reasonable cooling factor of 2 is chosen as the definition of a cooling stage, then the equilibrium emittance and, correspondingly, the beta at the absorber could be scaled up by a factor of 2 per stage relative to the final one. Since chromaticities and other related optical aberrations are a strong function of focusing strength (which is required to achieve the low-beta functions), relaxing the low-beta conditions impacts tremendously the design, stability and strength of the elements used in the upstream cooling stages.

When beta functions at the absorber exceed or approach one meter, the focusing strength is dramatically reduced and the absorber no longer has to be located at the lowest-beta point, allowing more flexibility in the choice of optical structure and focusing elements. Such a channel cools efficiently for rms beam emittances (normalized) larger than 6.4 mm rad. This observation represents the motivation for designing a competitive cooling channel based on normal-conducting quadrupoles in a simple lens, or FODO-cell configuration, a channel style that is maintainable for at least the first two stages of cooling (assuming a factor of 2 cooling per transverse plane in each stage).

5. FODO-based quadrupole cooling cell

With the relaxed beta at the absorber, one can consider a short, alternating quadrupole lens structure. The advantages of a short FODO cell structure over a doublet or triplet quadrupole telescope are primarily in the acceptance and stability of optical parameters over a tremendous chromatic range. The dynamical range in telescope structures is about $\pm 5\%$ $\delta p/p$, beyond which the off-axis transverse orbits are not stable in a repeating structure. The limited momentum acceptance of the triplet/doublet quadrupole channels restrict their implementation to after longitudinal, or momentum, cooling has occurred and are not considered further here. However, in standard (implying repetitive) FODO-cell optics, the minimum beta in one plane is located at the maximum beta in the other. A minimum beta or beam size cannot be established simultaneously in both planes, and, therefore, the absorber cannot be located at the lowest-beta point in this type of channel. The smallest beta for both planes combined is found halfway between the quadrupoles, at the "crossing point" in β_x and β_y. Due to this limitation, an effective application of a FODO-based cooling channel is just after capture and phase rotation and represents the first, or first two stages, in the cooling process. Our definition of cooling stage is that proposed here: a reduction factor of 2 per plane from the initial transverse emittances.

For a short FODO cell, the average beta in both planes is equal and lies between 1 and 2 m for normal conducting quadrupoles and limited (~ 0.5 m) spacing between them. The value of the beta functions at the crossing point is unusually stable over a large momentum range: from –20% to almost $+100\%$ if the phase advance is adjusted properly. The optics rationale for its design and stability will be discussed after a presenting the physical parameters chosen for this channel.

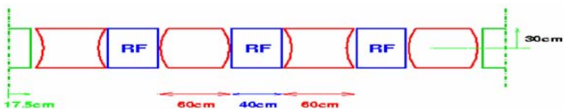
Initially, the physical design parameters were chosen to be competitive with a solenoidal precooler and the beginning sFOFO channel with starting and end emittances of 20.4 and 10.2 mm rad (rms), respectively. This final emittance requires a design equilibrium emittance of 6.8π mm rad for efficient cooling, which corresponds to an average beta at the absorber of 1.6 m. The optical and physical parameters for a channel that satisfy this constraint on beta at a central momentum of 200 MeV/c (as in the sFOFO design) are shown in Table 1 along with a rough channel layout.

The aperture of the quadrupole was chosen somewhat conservatively in order to constrain its length to be equal to its aperture in order that the quadrupole field profile and therefore the optics are not fringe-field dominated. This aperture is not quite sufficient to accommodate the proposed 2.5σ for an initial rms emittance of 20.4π mm (it needs to be closer to a 40 cm radius), although this larger acceptance can be achieved to some extent using star-shaped vacuum chambers that extend between the quadrupole poletips.

With the extreme demands placed on momentum performance, it is instructive to examine the FODO cell under the precepts of thin-lens conditions. First, it is useful to choose a reference momentum, p_0, and study the phase advance as a function of momentum relative to the reference in order to evaluate performance limits. For such a study, it is only practical to assign a working point, or initial cell phase advance, to this reference momentum and one which is centrally located between stability limits: 0° and 180°. Clearly 90° is an obvious choice, hopefully optimizing the momentum reach of the channel. This choice of phase advance was applied to a p_0 of 200 MeV/c, a value chosen to be compatible with current cooling channel designs, which is the sFOFO central momentum. The phase advance dependence, φ, on momentum is first obtained beginning with the thin-lens approximation:

$$\sin\frac{\varphi}{2} = 1/\varsigma; \quad \text{where} \quad \varsigma = f/L \text{ (thin lens).} \tag{2}$$

Table 1
Physical parameters for the quadrupole cooling channel chosen to match the sFOFO channel of Ref. [2]



Here, f is the focal length of $\frac{1}{2}$ of a full quadrupole, and L is the length of a half cell from quadrupole center to center (see, for example references listed in Ref. [7]). Since $f = (kl)^{-1}$ with kl being the *strength*, $(\mathrm{m}^{-2}) \times$ *length* (m) of a half-size quadrupole this equation can be modified.

$$\sin\frac{\varphi}{2} = \frac{0.3B'l}{p}L; \quad \text{since } k = \frac{0.3B'l}{p}. \tag{3}$$

In Eq. (3), B' is the quadrupole gradient in T/m and p is the momentum in GeV/c. Selecting $\varphi = 90°$ at p_0, the reference momentum implies the following:

$$\sin\frac{\varphi}{2} = \frac{1/\sqrt{2}\,p_0}{p}, \text{ with a clear lower limit of stability,}$$
$$p = \frac{1}{\sqrt{2}}p_0. \tag{4}$$

Differentiating the above equation gives the dependence of phase advance on momentum.

$$\frac{1}{2}\cos\frac{\varphi}{2}\,\mathrm{d}\varphi = -\frac{p_0}{\sqrt{2}p^2}\,\mathrm{d}p \tag{5}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}p} = -\frac{\sqrt{2}p_0}{p^2\sqrt{1 - p_0^2/2p^2}}. \tag{6}$$

Notice that for a p_0 of 200 MeV/c, the above analysis (Eq. (4)) gives a lower momentum cutoff for the channel of ~140 MeV/c and, at large p, the phase advance varies more and more slowly, as $1/p^2$. The results of this analysis are graphed in Fig. 3 clearly demonstrating the large play in momentum of the simple-lens FODO cell. When compared with calculations, an almost constant factor of 0.8 was needed to translate the changes in phase advance from the thin-lens model to ones accurate for the channel as designed. "Thick" quadrupoles actually extend the momentum reach of the channel beyond the thin-lens prediction.

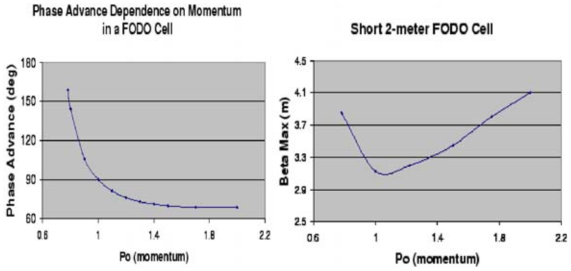


Fig. 3. On the left is the phase advance of a FODO cell plotted with respect to an arbitrary momentum, p_0, whose phase advance has been set to 90°. On the right is the variation of the peak beta function relative to p_0 for a half-cell length of 1 m.

The slow variation in phase advance does not set restrictions on the length of the cell, but the variation of the peak beta function with momentum does. Using the definitions above, the peak beta function for a FODO cell is given by

$$\beta_{\max} = L\frac{\varsigma(\varsigma+1)}{(\varsigma^2-1)^{1/2}} \tag{7}$$

$$\frac{d\beta_{\max}}{dp} = L(\varsigma^2-\varsigma-1)\frac{d\varsigma}{dp} = 0 \text{ for a minimum.} \tag{8}$$

In the above Eq. (8), $(\varsigma^2-\varsigma-1)$ can only be set to 0 locally (at ~76°), but this does not guarantee stability in the beta function over a large range in momentum. The only approach that minimizes $d\beta_{\max}/dp$ over a broad spectrum is to let L approach 0. No drift between quadrupoles is optimal, but the choice of a short drift of ~0.5 m (which corresponds here to a half-cell length of 1 m) intentionally slows the variation of the maximum beam size with energy and at the same time insures a more feasible technical channel design. (Absorbers and RF cavities are not installed inside magnet apertures.) The variation of the maximum beta with momentum for this design is shown below.

Table 2 summarized the important optical parameters of this channel as related to cooling. Of particular importance is the stability of the peak beta for a short FODO cell. When the momentum dependence of the average beta at the absorber was studied, the observed change was also small up to 300 MeV/c due to the slowly varying peak beta values. (The beta function dependence is approximately linear across a cell; this implies the beta function at the absorber is approximately $\frac{1}{2}(\beta_{\max}+\beta_{\min})$ making the peak beta the most important contribution.) Although the minimum beta value is a strong function of energy, the stability of the peak beta also indicates the large geometrical acceptance will remain constant as a function of energy for a given quadrupole aperture.

Table 2
Catalog of the important optical functions for the quadrupole cooling cell as a function of momentum

P (MeV/c)	β at absorber (m)	ν per cell ($\times 2\pi$ rad)	β max (m)	β min (m)
155	1.57	0.33	3.85	0.35
200	1.63	0.23	3.13	0.71
245	1.91	0.18	3.20	1.10
300	2.28	0.15	3.45	1.37

Constancy of the peak beta function implies that the physical dimension and divergence of the beam is maintained independent of energy; i.e. a constant geometrical admittance. By contrast, in the solenoidal channel, the normalized admittance is constant because the beta function increases with beam energy (also indicated by a rapid change in phase advance). As a result of rising beta functions the solenoidal channel displays a sharp cutoff at 245 MeV/c due to physical not dynamical apertures. Because of the sharp rise as a function of momentum, increasing the solenoid's apertures further (they are already comparable to the quadrupole bores), does not significantly improve the momentum performance. From the table it is evident that the quadrupole channel is still responsive at 300 MeV. The exact cooling limits of the channel will be explored later in this work (Fig. 4).

6. Simulations

Simulations were performed with the program COSY [8] on the quadrupole cooling cell described

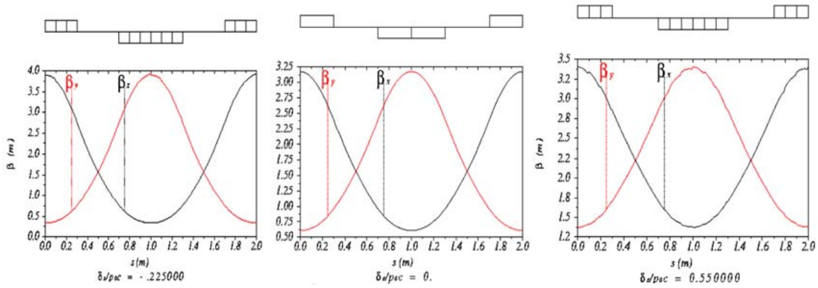


Fig. 4. Plots of the beta functions for the quadrupole cooling cell at $p = 155$, 200 and 300 MeV/c.

in the last section:

- using full nonlinear terms;
- with/without quadrupole fringe fields [9,10] (including different models);
- including multiple scattering (hydrogen absorber + windows);
- energy loss, dE/ds, as a function of energy;
- full energy loss simulation: straggling and spin;
- a 200 MHz sinusoidal RF cavity.

6.1. Fringe fields

Simulations were initiated using a standard Enge function fall-off [11] for the quadrupole end-field profile which is known to represent fringe-field effects to an accuracy sufficient for the initial analysis:

$$F(z) = \frac{1}{1 + \exp(a_1 + a_2(z/D) + \cdots a_6(z/D)^5)} \qquad (9)$$

where z is the distance perpendicular to the effective field boundary (for multipoles this is the arc length along the reference trajectory), D is the full aperture of the element and $a_1 - a_6$ are the Enge coefficients which depend primarily on the geometric details of the element and are adjusted to reproduce measured or design-computed data. The simulations were performed with "as-built" or measured quadrupole end fields of existing large-aperture quadrupoles. Examples of the fringe-field fall off used in two of the simulations are given in Fig. 5.

The fringe-field impact on the optics is strong and shifts in the phase advances had to be compensated by changing the quadrupole field

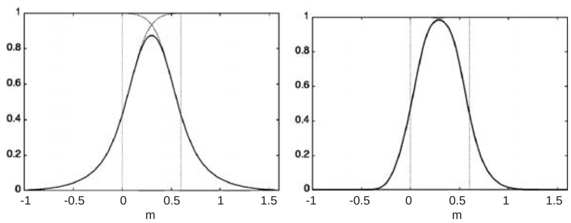


Fig. 5. Two examples of fringe-field models used in the simulation: Enge-falloff function for PEP/SLAC quadrupole (right) and for MSU/S800-Q2 (left). Profiles were adjusted to reflect the 60 cm in length × 60 cm in aperture requirements.

strength. Particles were tracked along increasing invariant ellipses in 2 cm steps to determine the dynamic aperture. Without fringe fields, particle motion is qualitatively linear and the dynamic aperture is effectively unlimited outside of limited resonance conditions. (At an energy where the cell phase advance is 90° notable resonance islands developed in the presence of kinematical effects alone due to the large emittances considered. It was interesting to further note that fringe fields actually mitigate the development and stability of resonance islands.) With fringe fields the ellipse changes and the consequences of different models are cataloged in the following series of phase-space plots (Fig. 6). It is important to note that the plots are identical for x, x' and y, y' due to the complete symmetry of the system in x and y and the absence of nonlinear elements such as sextupole correctors.

However, once the linear tune is adjusted to the design value in the presence of fringe fields, the acceptance of this channel, even in the absence of cooling and stochastic processes was still large and comparable to the sFOFO channel; although the acceptance increased strongly with energy, with

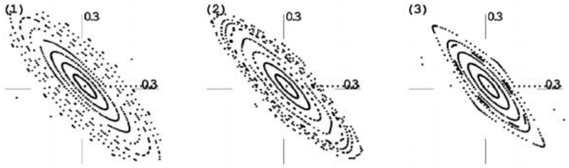


Fig. 6. Tracking of particle motion (EXPO method [10]) in phase space with COSY in the quadrupole cooling channel using (1) standard COSY default (Enge fringe fields), (2) PEP/SLAC fringe fields of Fig. 5, and (3) MSU/S800-Q2 fringe fields of Fig. 5. The point at the ends of each axis mark represents 0.3 m horizontally and 0.3 rad vertically.

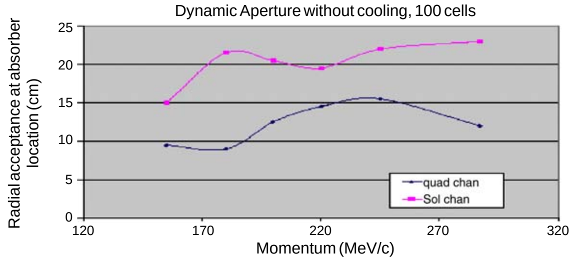


Fig. 7. Radial (not emittance) acceptance at location of the absorber in quadrupole and solenoidal channels without cooling, but fringe fields in the case of the quadrupole. The top line is the solenoidal channel and the bottom the quadrupole.

the opposite being true of the solenoidal acceptance. As can be seen from the tracking pictures, the elliptical nature of the particle motion suffers little distortion in the presence of these fringe fields indicating that the kinematics are predominately governed by linear equations of motion; hence the persistently large dynamic aperture of the quadrupole channel. The dynamic apertures of the basic magneto-optics for both an ideal solenoidal sFOFO channel and this quadrupole channel, including full fringe fields, is given in Fig. 7 exclusive of non-symplectic effects due to the absorbers and RF cavities. Particles were launched and tracked for 100 cells. The figure does not correspond to actual emittance accepted by the solenoidal channel because the beta functions are rising rapidly with energy thereby constricting the admittance. Even in the presence of nonlinear fringe fields the quadrupole channel was found to perform well compared to a "perfect" solenoidal channel, one with ideal, carefully manicured field profiles. Once can deduce from this that the quadrupole channel, as designed, is unusually insensitive to field errors as a direct consequence of the slowly changing optical parameters with energy.

6.2. Absorber and RF cavity: simulating discrete energy jumps

More challenging than fringe-field characterization, was to describe and quantify the performance of the quadrupole channel including cooling and multiple scattering; first the question was posed as to whether such a channel could be evaluated using the conventional elliptical description of the phase space, or whether the elliptical nature of phase space, implying linear particle motion, was not preserved in the presence of discrete energy steps and multiple scattering. Discovery of an invariant ellipse was found despite the large, discrete energy jumps due to de-acceleration in the absorber and re-acceleration in the RF cavity through a simple formalism. The discrete energy jumps produce a determinant in the transfer matrix which is different than 1. A simple renormalization of the determinant back to unity by scaling by the damping factor $\sqrt{D}$ (where D is the determinant of the linear map of the decoupled, damped system) eliminates particle travel between concentric ellipses and confines particle motion to a single ellipse, one that characterizes the system. The effect of renormalizing is illustrated in Fig. 8. This ellipse has a slightly different aspect ratio and orientation from the pure magnetic-optics ellipse without the absorber and RF cavity. If particles are launched on the ellipse found empirically from this renormalization (which is easily determined by tracking a single particle through multiple channels), then the cooling process simply collapses this ellipse symmetrically about the perimeter to ones of lower phase space area; i.e. eventually reaching the equilibrium ellipse. The dynamic aperture is then completely conventional: it is the largest phase space ellipse that survives in the presence of cooling (which is significantly larger than in the absence of cooling). In all studies of dynamic aperture with cooling fringe fields were included.

Fig. 8. Tracking via the damped transfer map and tracking via the transfer map with renormalized determinant. The end of the axis mark in each figure is 0.5 m (horizontal) and 0.5 rad (vertical).

6.3. Multiple scattering

Multiple scattering clearly extends the phase space ellipse in the x' direction, also changing the aspect ratio of the invariant ellipse, but not the orientation. This change is not at present considered in the channel design since, in principle, it is small for low values of the beta function and a hydrogen absorber. However, since the channel is not "rematched" for multiple scattering, eventually the area of the invariant ellipse would increase through filamentation by nonlinearities (the fringe fields). It has been found that the cooling rate is so rapid, that multiple scattering "fills in"—or diffuses to the interior of the invariant ellipse found for cooling, but the process does not have time to filament significantly and blow up the ellipse unless the channel is operated near the equilibrium emittance.

7. Simulation results

Since the cooling rapidly reduces the emittance (cooling by a factor of two per plane in the current design), the dynamic aperture is almost not relevant because a beam that fills the entire quadrupole aperture is cooled and is not lost even in the presence of fringe fields. In fact, it is the cooling acceptance in momentum that may be the deciding design criterion for cooling channels (Fig. 9).

As mentioned it is important to calibrate the expanded momentum reach of the quadrupole cooling channel. The important point to note is that because the geometrical acceptance is constant, the normalized one is increasing (the relativistic velocity is not changing significantly). Hence, the absorber beta can be allowed to track the increase in normalized emittance acceptance. When benchmarked against the full cooling simulation performed at a p_0 of 200 MeV/c, the channel still cools at 400 MeV/c. From the plot in

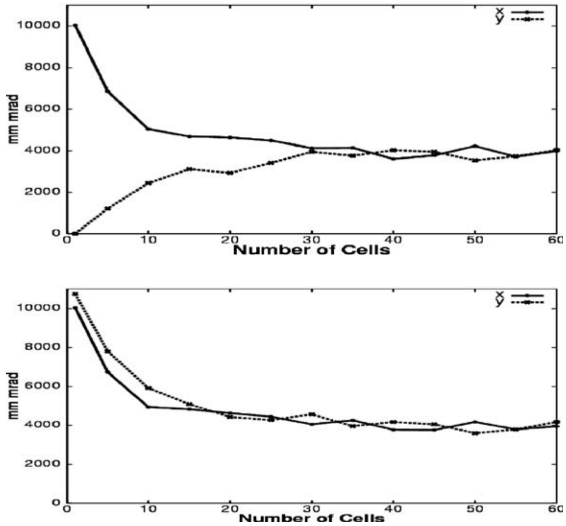


Fig. 9. Emittance reduction as a function of number of cooling cells. Top plot shows this starting emittance in x, no emittance in y with corresponding emittance growth up to the equilibrium emittance. In the bottom plot are particles launched in x, y Gaussian distributions, but along the diagonal.

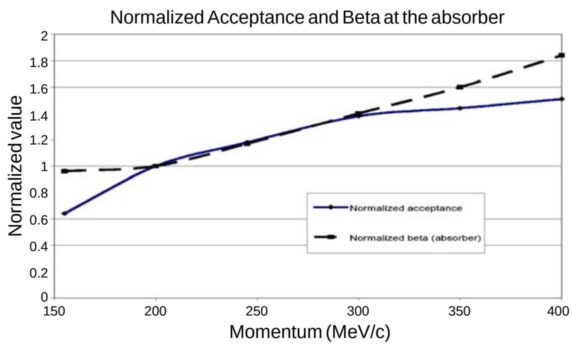


Fig. 10. The normalized emittance acceptance of the quadrupole channel divided by the emittance acceptance at 200 MeV/c is plotted as a function of momentum (solid line). The beta at the absorber divided by the beta at 200 MeV/c (1.6 m) is also plotted for comparison versus momentum.

Fig. 10 it is observed that the slopes of the normalized emittance acceptance and the beta at the absorber are close as a function of momentum.

8. Conclusions

Due to the short muon lifetime and rapid acceleration times required, RF represents by far the dominant cost and power consumption in both a Neutrino Factory or a Muon Collider. It is important to optimize the muon collection and intensity given the large cost to accelerate muons. The unavoidable consequences are ultra-large components in the initial stages of the facilities and stability over unprecedented emittances to lower or optimize the cost per muon.

Here, a linear (both geometrically and optically) cooling channel constructed from the simplest quadrupole lens system has successfully demonstrated ionization cooling suitable for either a Neutrino Factory or a Muon Collider. The relaxed beta function in such an optiical structure at the absorber limits application of such a channel to the large-emittance domain following muon production and capture. Detailed simulations show that the channel cools efficiently and much beyond the momentum range of a sFOFO [2] cooling channel with similar magnetic apertures, or from approximately 155–400 MeV/c in the former compared with 155–250 MeV/c in the latter. Applying different—both assumed and measured fringe fields—to represent the quadrupole elements fully has been an integral part of the simulations and ensures feasibility in quadrupole design and performance. An important concluding observation is that such a channel cools effectively over a large variation in the fringe-field profile.

The relative insensitivity of the optics to energy indicates not only stable performance, but also a large tolerance of field and alignment errors. The importance of straggling in emittance re-heating is also greatly reduced through this insensitivity. Further, amplitude-pathlength, and transverse–longitudinal correlations, yet another source of strong nonlinear performance and effective emittance increase is absent in the quadrupole channel.

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