

# THE ADVANCEMENT OF COOLING ABSORBERS IN COSY INFINITY\*

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## Abstract

COSY Infinity is an arbitrary-order beam dynamics simulation and analysis code. It can determine high-order transfer maps of combinations of particle optical elements of arbitrary field configurations. For precision modeling, design, and optimization of next-generation muon beam facilities, its features make it a very attractive code. New features are being developed for inclusion in COSY to follow the distribution of charged particles through matter. To study in detail some of the properties of muons passing through material, the transfer map approach alone is not sufficient. The interplay of beam optics and atomic processes must be studied by a hybrid transfer map–Monte-Carlo approach in which transfer map methods describe the average behavior of the particles in the accelerator channel including energy loss, and Monte-Carlo methods are used to provide small corrections to the predictions of the transfer map accounting for the stochastic nature of scattering and straggling of particles. The advantage of the new approach is that it is very efficient in that the vast majority of the dynamics is represented by fast application of the high-order transfer map of an entire element and accumulated stochastic effects as well as possible particle decay. The gains in speed are expected to simplify the optimization of muon cooling channels which are usually very computationally demanding due to the need to repeatedly run large numbers of particles through large numbers of configurations. Progress on the development of the required algorithms is reported.

## INTRODUCTION

Muons are tertiary production particles (protons → pions → muons) and high-intensity collection requires a large initial phase space volume. The resultant spray of muons must be amassed, focused, and accelerated well within the muon lifetime (2.2 μs in the rest frame). The only technique fast enough to reduce the beam size within the muon lifetime is ionization cooling. When muons traverse a material, both the longitudinal and transverse momentum components shrink due to ionization. The energy is then restored in the longitudinal direction only, leading to an overall reduction in the transverse beam size (cooling). In order to achieve cooling in the longitudinal direction, emittance exchange is used, usually involving wedge-shaped absorbers. For some applications such as a high-energy high-luminosity muon collider, cooling needs to be very aggressive: six-dimensional emittance reduction over six orders of magnitude is required to reach design goals.

In order to carefully simulate the effect of the absorbers on the beam, one needs to take into account both deterministic and stochastic effects in the ionization energy loss. The deterministic effects in the form of the Bethe-Bloch formula with various theoretical and experimental corrections fit well into the transfer map methods approach, where the effect of the lattice on the particles is evaluated first by producing the so-called transfer map, and then is applied to a given initial distribution of particles. The arbitrary-order simulation code COSY Infinity [1] is a key representative of transfer map codes. COSY was chosen because of its built-in optimization tools, speed, its ability to produce high-order transfer maps, and its ability to control individual aberrations.

However, to take into account stochastic effects the transfer map paradigm needs to be augmented by implementing the corrections from stochastic effects directly into the fabric of COSY. Some of the fundamental ideas of the process were presented in [2] in application to quadrupole cooling channels, but the approximations used were fairly basic. In this work, a more rigorous theoretical approach is presented along with the resulting validation.

## STOCHASTIC PROCESSES

The stochastic processes of interest are straggling (fluctuation about a mean energy loss), angular scattering, transverse position corrections, and time-of-flight corrections (corresponding to the longitudinal position correction). The general outline to simulate these four beam properties will be discussed and benchmarked against two other beamline simulation codes, ICOOL [3] and G4Beamline [4], and (in the case of angular scattering) against experimental data [5]. The simulation followed the beam properties cited in [5], which were a pencil beam with an initial momentum of 172 MeV/c through 109 mm of liquid hydrogen (LH) with cylindrical geometry. The step sizes for ICOOL and G4Beamline were chosen to be a modest 1 mm in order to ensure a quality simulation. The step size for COSY was chosen as the entire cell (109 mm), since its algorithms are largely insensitive to step sizes, as will be shown later.

### Straggling (Figure 1)

As the momentum range of interest is 50–400 MeV/c through low-Z materials, only ionization effects contribute to the mean energy loss. As such, Landau theory accurately describes the energy loss spectra, having the form [6]

$$f(\lambda) = \frac{1}{\xi} \cdot \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \exp(x \ln x + \lambda x) dx,$$

where  $\xi \propto Z\rho L/\beta^2 A$ , and  $\lambda \propto dE/\xi - \beta^2 - \ln \xi$ . Here,  $Z$ ,  $A$ , and  $\rho$  are the material parameters of charge, atomic mass,

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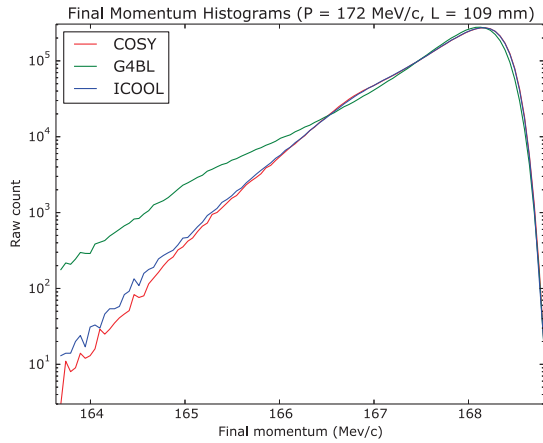


Figure 1: Straggling comparison between COSY (red), G4Beamline (green), and ICOOL (blue).

and density;  $L$  is the amount of material that the particle traverses;  $\beta = v/c$ ; and  $dE$  is the fluctuation about the mean energy.

### Angular Scattering (Figure 2)

The derivation of the scattering function  $g(u)$  (where  $u = \cos\theta$ ) is done separately for small angles and large angles. For small angles, the shape is very nearly Gaussian in  $\theta$  [7]. For large angles, the distribution follows the Mott scattering cross section, and is Rutherford-like [8]. The resulting peak and tail are continuous and smooth at some critical  $u_0$ , which yields the final form of  $g(u)$

$$g(u) = \begin{cases} e^{-\frac{1}{2} \frac{1-u}{1-u_\sigma}} & | u_0 < u \\ \zeta \cdot \frac{1 + \frac{1}{2}(\beta\gamma)^2(1+u-b)}{(1-u+b)^2} & | u \leq u_0 \end{cases}$$

Here the parameters  $\zeta$  and  $b$  are chosen to ensure continuity and smoothness. The familiar terms take their usual meaning:  $\beta = v/c$  and  $\gamma = 1/\sqrt{1-\beta^2}$ .  $u_0$  is a fitted parameter, and was chosen as  $u_0 = 9u_\sigma - 8$ .  $u_\sigma$  is the  $\sigma$ -like term for a Gaussian in  $\theta$ . It is another fitted parameter and takes the form

$$u_\sigma = \cos\left(\frac{13.6 \text{ MeV}}{\beta pc} \left(\frac{L}{L_0} \left(1 + 0.103 \ln \frac{L}{L_0}\right) + 0.0038 \left(\ln \frac{L}{L_0}\right)^2\right)^{\frac{1}{2}}\right)$$

### Transverse Position Corrections (Figures 3 and 4)

Since there occur multiple scatterings in a given medium, one must take into account the transverse position correction. A good starting point for these considerations is in [9]. If the scattered angle  $\theta$  is known then the transverse displacement correction is generated from a Gaussian distribution with

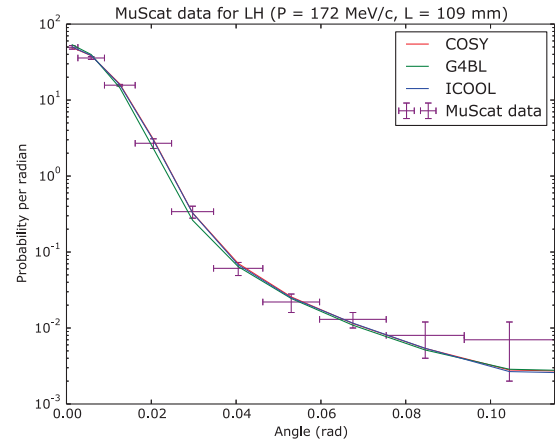


Figure 2: Angular scattering comparison between COSY (red), G4Beamline (green), ICOOL (blue), and the data points from [5] (purple).

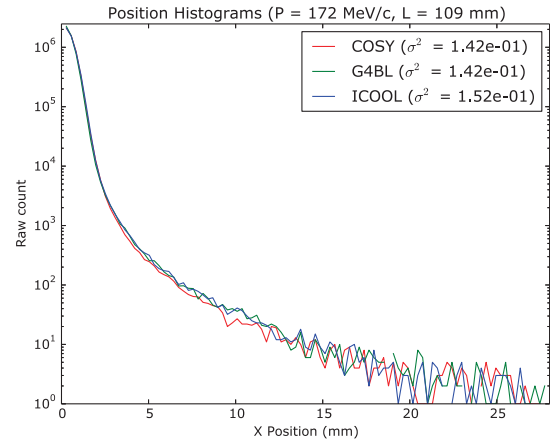


Figure 3: Transverse position comparison between COSY (red), G4Beamline (green), and ICOOL (blue).

mean  $\mu_T$  and standard deviation  $\sigma_T$ . These are chosen as

$$\mu_T = \frac{\theta \rho_c L}{\mu_w}, \quad \sigma_T = \max\left(L\theta_\sigma \sqrt{\frac{1-\rho_c^2}{3}}, \left|\frac{LP_T/P_Z}{\sigma_w}\right|\right),$$

where  $\rho_c = \sqrt{3}/2$  is the correlation coefficient,  $\theta_\sigma$  corresponds to the aforementioned  $u_\sigma$ ,  $P_T$  and  $P_Z$  are the particle's transverse and longitudinal momenta, and  $\mu_w = 1 + \sqrt{3}/2$  and  $\sigma_w = 6$  are adjustable parameters. It should be further noted that  $\mu_T$  must be given the proper sign, i.e. the same sign as the desired transverse momentum. Additionally, this fluctuation assumes an initially straight trajectory in the lab frame, and hence must be rotated accordingly and added to the mean (deterministic) transverse position deflection.

Perhaps more important than the raw histogram is the transverse phase space. This is because, for example, the raw histogram is insensitive to the  $\sigma_T$  changes, which describe

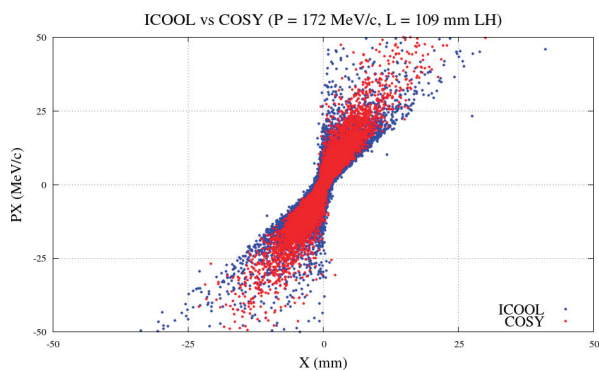


Figure 4: Transverse phase space comparison between COSY (red) and ICOOL (blue).

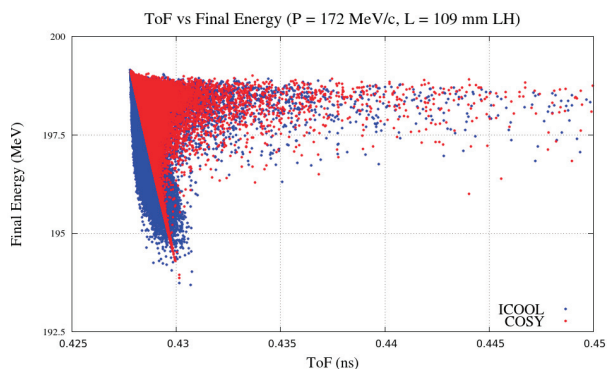


Figure 5: Longitudinal phase space comparison between COSY (red) and ICOOL (blue).

the transverse position spread given a particular scattered angle.

### Time-of-Flight Correction (Figure 5)

When particles traverse matter, the deterministic ‘straight’ path length differs from the ‘true’ path length due to many multiple scatterings within the material. The cases of straggling, angular scattering, and transverse position correction are largely insensitive to this. However, as the time-of-flight for these purposes is on the order of 1 ns for a single absorber, the true pathlength correction must be taken into account. Ref. [10] gives a good approximation to the true path length  $t$  given the straight path length  $L$  and the scattered angle  $\theta$ :

$$t = \frac{2L}{1 + \cos\theta}.$$

Similar to the transverse position, the time-of-flight corrections have important implications for the overall shape of the longitudinal phase space.

## CONCLUSIONS

The addition of stochastic processes in COSY Infinity for the use of muon ionization cooling has been largely successful. While the straggling data in Figure 1 agrees well with ICOOL, there is some discrepancy in the tail when compared to G4Beamline. This may be due to several factors which can be found in the physics reference manual of [11]. For

example, the straggling model of [11] takes into account the cross sections for ionization and for excitation, whereas the Landau theory used in COSY only regards the ionization cross section. Moreover, [11] uses a synthetic width correction algorithm to the curve, which is not elaborated in detail in the manual. For future improvements, it is expected that COSY will use the more general Vavilov theory [12], which converges to Landau theory for large energies or low absorber lengths.

The angular scattering algorithms appear to be functioning properly, as seen by Figure 2. While not shown, it is reported here that good agreement has been achieved between COSY and other sets of data from [5] (e.g. 109 mm of liquid hydrogen, 3.73 mm of beryllium).

The transverse position histogram in Figure 3 also shows that the algorithms in place appear to be largely in agreement with both ICOOL and G4Beamline. However, the phase space plot in Figure 4 shows that COSY appears to be narrow. This may be misleading, since the discrepancy is not between the bulk of the data but rather the lengthy tails. A better parameterization of  $\mu_T$  and  $\sigma_T$  may be necessary.

Similarly, the longitudinal phase space appears to agree fairly well between COSY and ICOOL. The discrepancy is on the order of 0.005 ns (roughly 1% of the mean time-of-flight). It is the opinion of this paper that while the agreement is good, there can still be improvements made, possibly in the approximation of true path length.

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