SEARCH FOR THE OPTIMAL SPIN DECOHERENCE EFFECT IN A QFS LATTICE

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on behalf of the JEDI Collaboration

Abstract

Measurement of electric dipole moment (EDM) in a storage ring requires the spin decoherence in a particle bunch to be less than 1 rad in 1000 s, which corresponds to about 1 billion turns. The quasi-frozen spin (QFS) method [1] has been proposed for deuteron EDM search. In a QFS lattice, spin direction turn in magnetic bend sections is compensated by spin direction turn in electrostatic bend sections, and thus the spin direction at a point in the lattice is approximately constant. We consider two QFS lattices, each with an RF cavity and a family of sextupoles. In *COSY INFINITY*, calculations were done using transfer maps of the 7th order, with symplectic tracking using the Extended Poincaré (EXPO) generating function and the most accurate *COSY INFINITY* fringe field mode. We have optimized the sextupole strengths to minimize the spin decoherence. Using these sextupole strengths, we have done spin tracking of the lattice and analyzed the growth of spin decoherence as a function of the number of turns. Within their scope, our results indicate the feasibility of the QFS method.

INTRODUCTION

Frozen spin method has been previously proposed for EDM search. It is characterized by the spin vector of particle being aligned with its momentum while the particle moves within the lattice. The radial electrostatic field effects torque on the spin, causing it to rotate out of the midplane.

Quasi-Frozen Spin Method and Lattices

In quasi-frozen spin (QFS) method, the average projection of the spin vector on the momentum vector is approximately one. The mechanism for detection of the EDM is the same in QFS, but the implementation becomes simpler and more flexible [2].

We recall the Thomas-BMT equation:

$$\frac{d\vec{S}}{dt} = S \times \left(\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM}\right)$$

where

$$\vec{\Omega}_{MDM} = \frac{e}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{E} \times \beta}{c}\right]$$

and

$$\vec{\Omega}_{EDM} = \frac{e}{m}\frac{\eta}{2}\left[\frac{\vec{E}}{c} + \beta \times \vec{B}\right]$$

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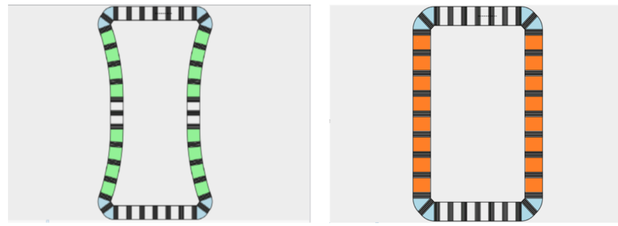

(a) *Senichev 6*. Structure includes 4 straight sections (light gray), 4 magnetic sections (blue), and 4 electrostatic sections (green). Lattice length is 16667 cm.

(b) *Senichev E+B*. Structure includes 4 straight sections (light gray), 4 magnetic sections (blue), and 4 E+B sections (orange). Lattice length is 14921 cm.

Figure 1: QFS Lattices.

The QFS condition is

$$\gamma G \Phi_B = \left[\frac{1}{\gamma}(1 - G) + \gamma G\right]\Phi_E$$

where Φ_B and Φ_E are the angles of momentum rotation in magnetic and electric bend parts of the ring correspondingly.

Yu. Senichev has proposed two QFS lattices, which we codenamed as *Senichev 6* and *Senichev E+B* [3](see Fig. 1). Both lattices operate with deuterons at the kinetic energy 270 MeV.

The RF cavity suppresses the 1st and partially the 2nd order of spin decoherence by mixing the particles relatively to the average field strength, and therefore, averaging out $\triangle\gamma G$ for each particle. Sextupoles are used to suppress the remaining 2nd order component, which is due to the average $\triangle\gamma G$ being different for each particle.

In *Senichev E+B* lattice, E+B static Wien filter elements are used instead of the electrostatic deflectors. This removes the respective nonlinear components and simplifies that lattice from the engineering perspective.

Fringe Fields of the Electrostatic Deflector

Senichev 6 lattice uses cylindrical electrostatic deflectors among other elements. Fringe fields of magnetic dipoles were previously studied extensively, but fringe fields of electrostatic deflectors were not studied in detail in the context of beam physics.

In 2015, we have calculated fringe fields of semi-infinite capacitors with solid metal plates in *MATLAB* using *Schwarz-Christoffel Toolbox* v.2.3 [4] and analyzed the results in *Mathematica*. In this analysis, we have compared the field falloffs with those of finite rectangular metal capacitors obtained in *Coulomb* by H. Soltner (IKP, Forschungszentrum Jülich, Germany).

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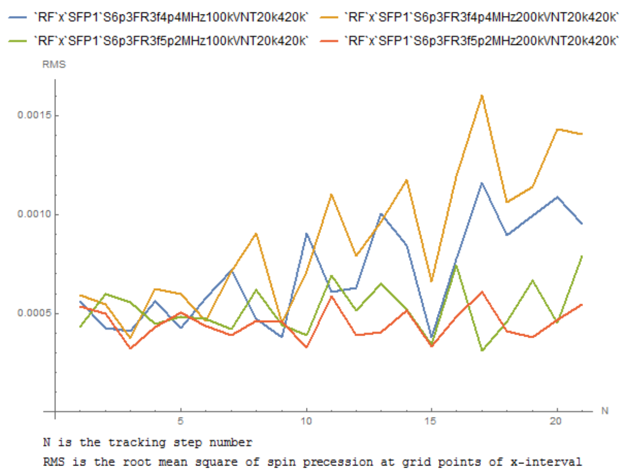
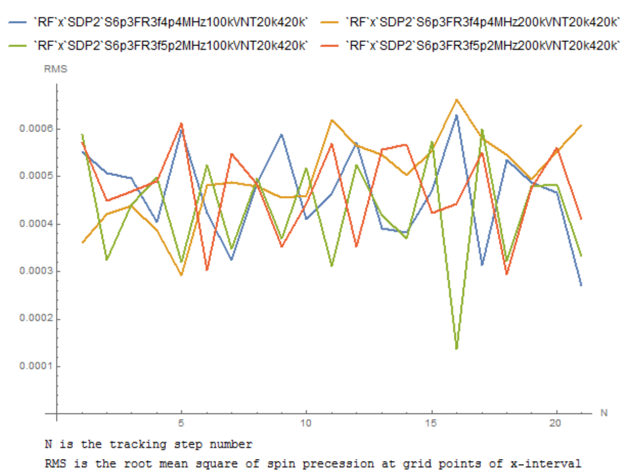
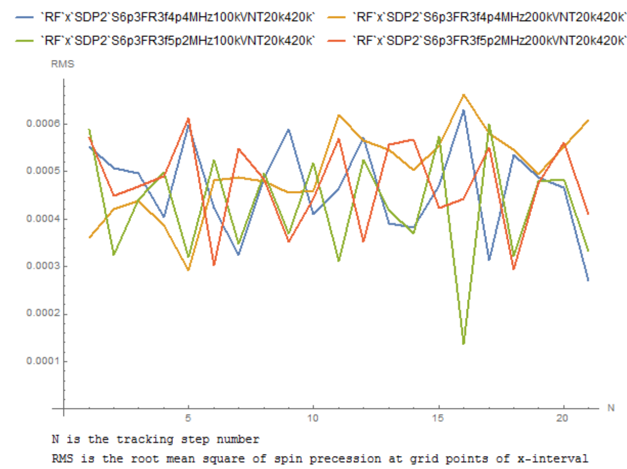


(a) Optimization by SFP1 sextupole family.

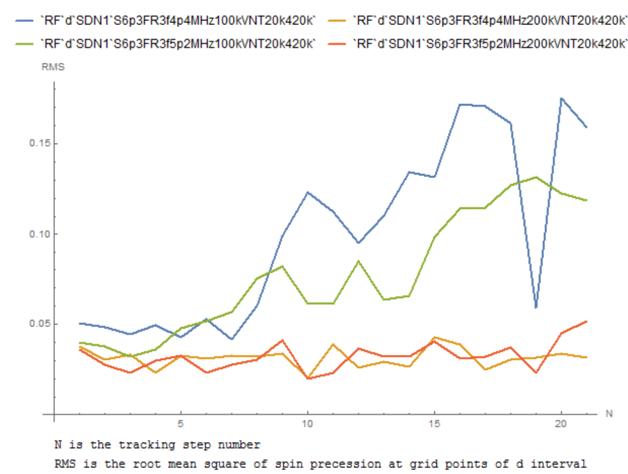


(b) Optimization by SDP2 sextupole famil.y

Figure 2: Spin decoherence in *Senichev 6* vs. number of turns up to 420k, FR 3 mode, $x - a$ plane.



(a) Optimization by SFP1 sextupole family.



(b) Optimization by SDN1 sextupole family.

Figure 3: Spin decoherence in *Senichev 6* vs. number of turns up to 420k, FR 3 mode, $l - \delta$ plane.

For *Senichev 6* lattice, we used Enge coefficients for a semi-infinite electrostatic deflector with rounded edges and with $d/10$ plate thickness, where d is the distance between a plate and the midplane (see Table 1).

METHODS

Analytic relations (quadratic, etc.) show the general character of the system. Numerical methods, such as system tracking in *COSY INFINITY*, give the final understanding of which orders are needed for spin decoherence to be less than 1 rad in 1000 s, i.e. 1 billion turns.

Table 1: Enge Function Coefficients for Electrostatic Deflectors in *Senichev 6* lattice.

Coefficient	Value
h_0	1.0614024399605924
h_1	1.6135741290714967
h_2	−0.9401447081042862
h_3	0.4781500036872176
h_4	−0.14379986967718494
h_5	0.017831089071215347

We have developed a solution, which consists of *COSY INFINITY* and *Mathematica* programs.

In *COSY INFINITY* programs, we manually and automatically optimize the sextupole strengths and perform long-term tracking of the resulting lattice. For effective energy averaging of particles, RF cavity frequency is set to a sufficiently high multiple of the $l - \delta$ motion frequency for effective energy averaging. Optimization and tracking data is saved in text and graphics files. Calculations are done using transfer maps of the 7th order. For tracking, the motion is symplectified using the Extended Poincaré (EXPO) generating function. Although it is considerably slower, we do final calculations in the most accurate fringe field mode FR 3.

The output of *COSY INFINITY* programs is used as input to the *Mathematica* programs, where data arrays are stored, processed, and QA checked. Context selector list is used to generate reports that consist of plots and tables. We analyze and compare the data using these reports.

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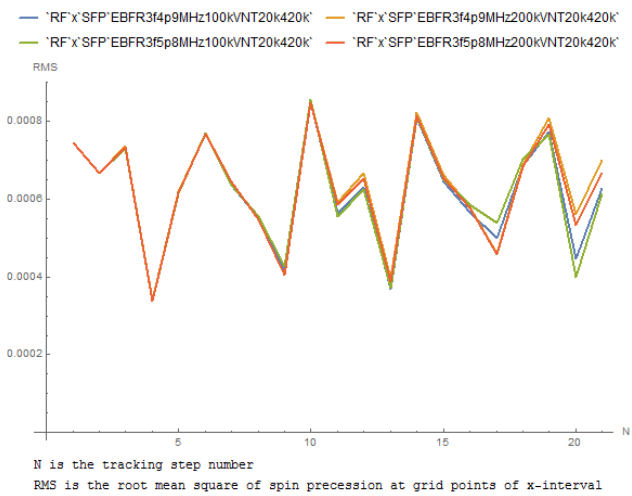
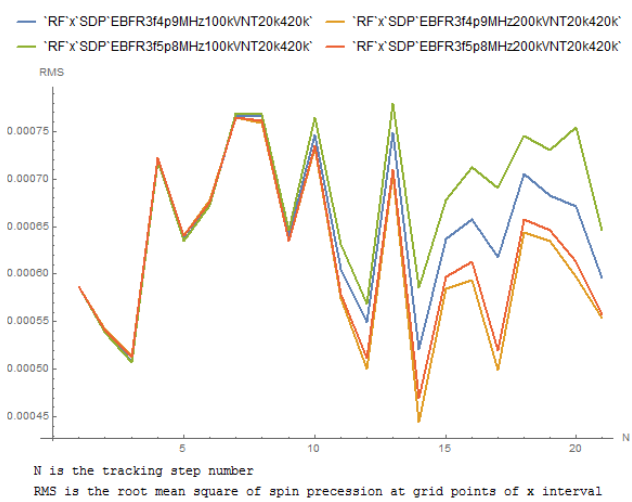


(a) Optimization by SFP sextupole family.



(b) Optimization by SDP sextupole family.

Figure 4: Spin decoherence in *Senichev E+B* vs. number of turns up to 420k, FR 3 mode, $x - a$ plane.

SUMMARY OF RESULTS

At 20 thousand turns, the order of spin decoherence in both systems was sufficiently large for 5×10^{-3} beam apertures in $x - a$ and $l - \delta$ planes of *COSY INFINITY*'s particle optical coordinate system [5] to require study of its longer term behavior. At present, we have studied its evolution up to 420 thousand turns in FR 3 mode and up to 1 million turns in FR 0 (no fringe fields).

With some input parameters (RF cavity frequency and voltage, sextupole settings), spin decoherence evolution has strong periodic minima, which can be explained by periodicity of RF modulation.

Spin decoherence evolution can be roughly classified into two types: (1) growing and (2) remaining in the same range. A necessary condition for the latter is the RF cavity being on.

With no fringe fields, spin decoherence as a function of a spacial coordinate is often bounded by two slanted lines.

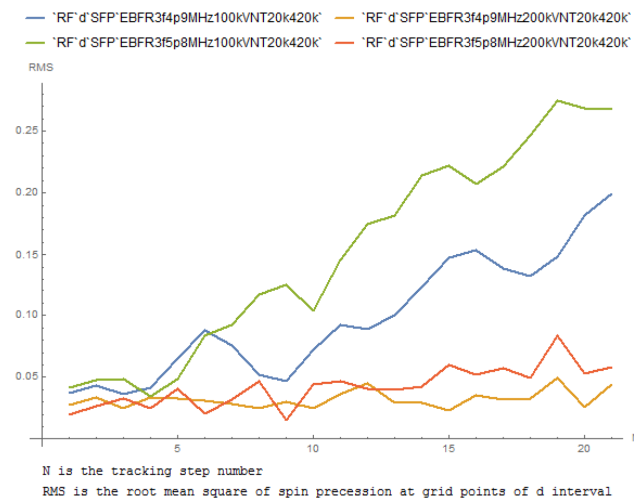


Figure 5: Spin decoherence in *Senichev E+B* vs. number of turns up to 420k, FR 3 mode, $l - \delta$ plane, optimization by SFP sextupole family.

We think that this illustrates the forced oscillations induced by the RF cavity. Turning fringe fields on distorts these lines to a various, but usually not very significant, extent.

DISCUSSION

Spin decoherence remaining in the same range for some parameters is indicative of the feasibility of the QFS method.

Spin decoherence should be optimized using the RF cavity and the sextupoles in all planes simultaneously. We may need to add octupoles to achieve spin decoherence of less than 1 rad in 1 billion turns. Tracking for more than 1 million turns is desirable for an optimal balance between computation time and confidence level, subject to any limits imposed by computational errors.

We will try to obtain a better optimization objective function by tracking a differential algebra (DA) vector-valued particle rays for multiple turns using modified *COSY INFINITY* tracking functionality.

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