Simulation and Optimization of the Tevatron Accelerator

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Summary. The Tevatron accelerator, currently the particle accelerator with the highest energy in the world, consists of a ring with circumference of four miles in which protons are brought into collision with antiprotons at speeds very close to the speed of light. The accelerator currently under development at Fermilab represents a significant upgrade, but experienced significant limitations during initial operation. The correction of some of the problems that appeared using techniques of automatic differentiation are described. The skew quadrupole correction problems are addressed in more detail, and different schemes of correction are proposed.

Key words: Tevatron, optimization, skew quadrupoles, COSY INFINITY, tracking, Fermilab, high-order multivariate AD, beam physics, flows, ODE's, transfer map, stability analysis

1 Introduction

The dynamics in a large particle accelerator is governed by relativistic equations of motion that are usually solved relative to those of a reference orbit. The simulation of an accelerator in this manner is a very demanding undertaking since particles orbit for in the order of 10^9 revolutions, and it is necessary to study many different orbits. Thus from the early days of particle accelerators, it has been customary to determine Taylor expansions of the flow, usually to orders two and three. Automatic differentiation methods, in particular in combination with ODE solving tools based on differential algebraic methods, have allowed us to increase this computation order very significantly, and now orders around 10 are routinely used in the code COSY INFINITY [38]. For the tracking pictures presented here, the order of calculation is usually taken to be seven for speed, but for some final results orders 11 or 13 are used. Furthermore, it is now possible to represent the devices by much more accurate models. A wide range of standard elements with the ability to simulate all nonlinearities and associated error fields is available in COSY INFINITY.

2 The Tevatron Accelerator – Machine Description

The Tevatron is currently the most powerful particle accelerator in the world with a circumference of the ring of four miles. The beams of protons and antiprotons moving in opposite directions are brought to collision at energies close to 1 TeV each. Hence, their relativistic kinetic energy is more than 1000 times that of their rest mass. Besides colliding two beams, it is necessary to make as many particles interact as possible. The effectiveness of the collision is characterized by a single value called luminosity. Calculating and optimizing the dynamics of the particle motion in the accelerator to reach higher and higher luminosity is one of the main goals of this work.

Particles in the Tevatron have velocities close to the speed of light, a fact which has several advantages and disadvantages for the modeling. COSY INFINITY takes all the resulting relativistic effects into account automatically.

The Tevatron consists of six arcs connected with six straight sections. Two of the straights are the well-known collision detectors CDF and D0. Each arc is a periodic structure having 15 FODO cells with 8 dipoles each to provide the necessary bending. However, each of these magnets has some error terms that are commonly called multipole moments. One of these errors is due to the fact that the coils of the dipoles are not parallel as they should be, which introduces a skew quadrupole term. This has a very detrimental effect on the motion of the particles, because it provides a coupling of the otherwise independent horizontal and vertical motion and thus affects the stability of the particles. A circuit of skew quadrupole correctors serves to compensate for these errors. The main problem addressed in this article is the scheme of such a correction and the view that only a part of the errors in the dipoles can be removed during the next Tevatron planned shutdown.

3 The Model, Criteria and Parameters to Control

We began with the basic model of the machine by V. Lebedev currently available at Fermi National Accelerator Laboratory [326–328] and converted the source code to run under COSY INFINITY. The tool converting the lattice description works automatically, so all the updates to the lattice easily can be taken into account. The current model implements elements: dipoles, quadrupoles, skew quadrupoles, sextupoles, skew sextupoles, solenoids with fringe field, and separators.

The recent upgrade of the Tevatron accelerator led to an undesirable coupling between the horizontal and vertical motion [202, 354, 491–494], while usually great care is taken to keep these two motions decoupled. Mere integration of orbits makes the task of decoupling very difficult, since it is very hard to assess from ray coordinates whether a coupling happens. On the other hand, in the framework of the Taylor expansion of final coordinates in terms of initial coordinates, decoupling merely amounts to

$$\frac{\partial x_k^{(f)}}{\partial y_j^{(i)}} = 0 \quad \text{and} \quad \frac{\partial y_k^{(f)}}{\partial x_j^{(i)}} = 0, \quad k, j = 1, 2 \tag{1}$$

for the respective linear effects, and to

$$\frac{\partial^{i_1+i_2} x_k^{(f)}}{\partial \left(y_1^{(i)}\right)^{i_1} \partial \left(y_2^{(i)}\right)^{i_2}} = 0 \quad \text{and} \quad \frac{\partial^{i_1+i_2} y_k^{(f)}}{\partial \left(x_1^{(i)}\right)^{i_1} \partial \left(x_2^{(i)}\right)^{i_2}} = 0, \quad k = 1, 2 \tag{2}$$

for nonlinear effects, where x_1 is the horizontal position, $x_2 = p_x/p_0$ is the reduced horizontal momentum, y_1 is the vertical position, $y_2 = p_y/p_0$ is the reduced vertical momentum, p_0 is some previously chosen scaling momentum; i_1, i_2 : $i_1 \geq 0$; $i_2 \geq 0$; $i_1 + i_2 < n$, n is the order of calculations, and the superscripts (i) and (f) denote the initial and final conditions, respectively.

At the same time, the main operating parameters of the machine, the two tunes (phases of the eigenvalues of the linear map), have to be kept constant. This is vital for the stability of the motion of the particles to help avoid resonances. In terms of partial derivatives, this condition amounts to the preservation of

$$\frac{\partial x_1^{(f)}}{\partial x_1^{(i)}} + \frac{\partial x_2^{(f)}}{\partial x_2^{(i)}} \quad \text{and} \quad \frac{\partial y_1^{(f)}}{\partial y_1^{(i)}} + \frac{\partial y_2^{(f)}}{\partial y_2^{(i)}} \; . \tag{3}$$

Thus with the availability of Taylor expansions, it is merely necessary to adjust suitable system parameters such that the ten conditions (in the linear case) described by (1) and (3) are met. While by no means an easy feat, this task is significantly more manageable than the attempt to optimize performance based on particle coordinates.

We are to control the strengths of several skew quadrupole correctors around the ring both in arcs and straight sections. Each arc has six to eight correctors, but it is preferable to have them all at the same strength as they have one power supply. Moreover, it would be advantageous to optimize them all to the same strength in all the arcs. The study shows that this can be done effectively and efficiently with AD methods implemented in COSY INFINITY. Without use of AD techniques all the calculations and especially optimization of the structure requiring intensive multiple recalculating of the transfer maps would take a prohibitively long time and could never be done for such a high order.

To keep the tune of the system constant, the strength of the main bus quadrupoles can be slightly changed. As all the quadrupoles have the same strength this task is a one-parameter optimization; besides it does not require high-order calculation. Even in this problem the high-order calculations are unavoidable, as checks should be made that changing the tune back to the original value after skew quadrupole correction optimization does not lead to degradation in the behavior of the particles described by the multi-revolution tracking picture.

4 Map Methods

The particles in most accelerators (the Tevatron is not an exception) usually stay close together, forming a beam. Therefore, it is convenient to choose a reference particle – the one that moves undisturbed through the centers of all the magnets of the machine and make use of perturbative techniques to obtain good approximation of the dynamics of motion of all the particles in the beam relative to this reference particle.

The motion of the particles is considered in 2-dimensional phase space: each particle has four coordinates x_1, $x_2 = p_x/p_0$, y_1, and $y_2 = p_y/p_0$, where p_0 is the momentum of the reference particle, and the arc length s along the center of the ring is used as an independent variable. Thus, the dynamics is described by the vector

$$\mathbf{z}(s) = \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix} ,$$

which depends on s. The action of the accelerator lattice elements can be expressed by how they change the components of the vector $\mathbf{z}(s)$. Denoting by $\mathbf{z}_0$ the initial coordinates at s_0, the final coordinates of each particle at s can be obtained from the system of equations

$$\mathbf{z}(s) = \mathcal{M}(s_0, s)\,(\mathbf{z}_0, \boldsymbol{\delta}) \ ,$$

relating $\mathbf{z}_0$ and a set of control parameters $\boldsymbol{\delta}$ at s_0 to $\mathbf{z}$ at $s > s_0$, where $\mathcal{M}(s_0, s)$ is a function which formally summarizes the action of the system. $\mathcal{M}$ is called the transfer function or transfer map of the system. The transfer map satisfies

$$\mathcal{M}(s_1, s_2) \circ \mathcal{M}(s_0, s_1) = \mathcal{M}(s_0, s_2) \ .$$

Therefore, the transfer map of the system can be built up from the transfer maps of individual elements. As the accelerator structure is regular, the set of different elements is not that big, a fact that allows us to seriously reduce the amount of calculations.

$\mathcal{M}$ is usually weakly nonlinear and can be considered as a sum of two maps: a purely linear part M, and a purely nonlinear part $\mathcal{N}$, i.e. $\mathcal{M} = M + \mathcal{N}$. For a simple analysis of the relative motion, often a linear approximation is enough, but for full understanding of the motion, the understanding of the nonlinear effects is essential.

Map methods are particularly useful for the study of the motion in circular accelerators, as one has to run the particles through the same system many times. The number of revolutions necessary to estimate the behavior of the particles in the Tevatron lies in the orders of $10^5 - 10^7$. Having a transfer map of one full revolution, one easily can perform repetitive tracking of the particles through the system.

In case of the first order of calculations, the transfer map can be represented as a matrix of coefficients:

$$\begin{pmatrix} x_1^{(f)} \\ x_2^{(f)} \\ y_1^{(f)} \\ y_2^{(f)} \end{pmatrix} = \begin{pmatrix} 2.0756 & 0.0023 & 0.1123 & 0.0047 \\ 63.3276 & 0.5497 & 1.1253 & 0.1065 \\ 0.0003 & 0.0021 & 1.4570 & 0.0223 \\ -3.2571 & 0.0834 & 87.8001 & 2.0272 \end{pmatrix} \begin{pmatrix} x_1^{(i)} \\ x_2^{(i)} \\ y_1^{(i)} \\ y_2^{(i)} \end{pmatrix} .$$

For higher orders the numbering and processing of the coefficient is much more sophisticated. As an example, we here show a piece of a high-order transfer map:

```
I  COEFFICIENT             ORDER EXPONENTS
1  -.7246219112151764        1    1 0  0 0
2  -.9122414947039915        1    0 1  0 0
3  0.1588622636737591E-07    1    0 0  1 0
4  -.5419413125727635E-09    1    0 0  0 1
5   603.2477358626691        2    2 0  0 0
6  -1149.461216254035        2    1 1  0 0
```

7	783.4790397865372	2	0	2	0	0
8	-23.83485869335665	2	1	0	1	0
9	50.34274836239869	2	0	1	1	0
10	-222.4936370747880	2	1	0	0	1
11	98.21519926034055	2	0	1	0	1
12	-73.88432808806901	2	0	0	2	0
13	149.0913750043664	2	0	0	1	1
14	-223.3498687001470	2	0	0	0	2
15	-71088.03707913239	3	3	0	0	0
16	164852.2891153482	3	2	1	0	0
...	....	.	.	.	.	.
325	-16401453090653.15	7	0	0	4	3
326	29169856173501.98	7	0	0	3	4
327	-42361303120466.99	7	0	0	2	5
328	54860608468975.03	7	0	0	1	6
329	-43643493779626.68	7	0	0	0	7

Each row describes one term of the Taylor expansion of final coordinates in terms of initial coordinates. The columns labeled "EXPONENTS" describe the exponents of each of the independent variables appearing in the respective term. The "ORDER" column contains the total order of the term, i.e. the sum of the exponents, and the first column lists the double precision coefficient belonging to the respective term. As an example, the sixth row describes the Taylor series term depending on the power one of variable 1 and the power one of variable 2.

The map coefficients are the results of integrating the equations of motion of the particles through different lattice elements: quadrupoles, sextupoles, solenoids. The built-in ODE solver in COSY INFINITY works with differential algebra vectors – coefficients of Taylor expansion for the coordinates of the particles, which achieves very high orders of computations even on somewhat slow machines.

For the optimization of the linear coupling, the linear map is sufficient, as (1) and (3) affect only the linear part of the map. For subsequent correction of the nonlinear effects as in (2), it is necessary to determine higher order Taylor expansions of the map. More on the work with map methods and differential algebra approaches can be found in [37, 40]. The optimization works the following way.

1. Choose a correction scheme with different skew quadrupole circuits settings;
2. Perform the two-stage optimization. The first stage removes the linear coupling in each of the six arcs (the objective function is the sum of the derivatives in the left hand sides of the expressions (1)). The second stage removes the coupling for the whole machine (the objective function is the sum of the derivatives in the left hand sides of the expressions (1)) and brings the tune back to its initial value (the objective function is the sum of the expressions
$$\frac{\partial x_1^{(f)}}{\partial x_1^{(i)}} + \frac{\partial x_2^{(f)}}{\partial x_2^{(i)}} - C_x \quad \text{and} \quad \frac{\partial y_1^{(f)}}{\partial y_1^{(i)}} + \frac{\partial y_2^{(f)}}{\partial y_2^{(i)}} - C_y \ ,$$
where C_x and C_y are some fixed values);
3. Perform high-order tracking to check the stability of the motion after optimization.

The qualitative results of the optimization for two different skew quadrupole circuits are presented in Sect. 5, the quantitative results are discussed in Sect. 6.

5 Different Optimization Schemes and Proposals

Currently, 85% of the dipoles in the Tevatron still have the above-mentioned coil displacement. As a result, skew quadrupole components act on the particles. A first study consisted of leaving all the errors in place, but moving some of the correctors. The results for such optimization scheme are not shown, because this scheme was only interesting as a starting point, since subsequently it proved impossible to move any of the skew quadrupole correctors. At the same time, the scheme shows that good correction can be performed even if all the errors stay the same.

More realistic are the schemes with some of the errors in dipoles fixed and all or only the part of the correctors in their places. The forecast says up to 50% of the dipoles can be fixed during the upcoming Tevatron shutdown. The high-order analysis helps to determine exactly what should be done, which dipoles to correct and what the strength of the correctors should be to achieve the most predictable particle behavior, avoid resonances, and decouple the motion.

Sector E scheme I

FODO 1	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX
FODO 2	D*2	FQ		D*2	D*2 FIX	DQ	D*2FIX
FODO 3	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX
FODO 4	D*2	FQ		D*2	D*2 FIX	DQ	D*2FIX
FODO 5	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX
FODO 6	D*2	FQ		D*2	D*2 FIX	DQ	D*2FIX
FODO 7	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX
FODO 8	D*2	FQ		D*2	D*2 FIX	DQ	D*2FIX
FODO 9	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX
FODO 10	D*2	FQ		D*2	D*2 FIX	DQ	D*2FIX
FODO 11	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX
FODO 12	D*2	FQ		D*2	D*2 FIX	DQ	D*2FIX
FODO 13	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX
FODO 14	D*2	FQ		D*2	D*2 FIX	DQ	D*2FIX
FODO 15	D*2	FQ	SQC	D*2	D*2 FIX	DQ	D*2FIX

Fig. 1. Correction scheme I, errors in dipoles fixed in each cell around defocusing quadrupole.

The first scheme layout is shown in Fig. 1 in the form of the description of one sector. All the other sectors look similar except for some dipoles that have been fixed before this study was initiated. This scheme proposes to fix skew quadrupole errors in the dipoles (D*2 FIX) on both sides of the defocusing quad (DQ). All the skew quadrupole correctors stay in their places (SQC). The errors in dipoles around focusing quadrupole (FQ) remain unfixed (D*2).

The results of the optimization are given in Figs. 2 – 5. The first two pictures show the phase portraits in x_1, $x_2 = p_x/p_0$ and y_1, $y_2 = p_y/p_0$ planes for particle trajectories before the optimization. The next two show the results after the optimization. The scale of each picture is $2.4 \times 10^{-3}\,m$ horizontal and 4.0×10^{-3} vertical. For stable particles the trajectories look like closed curves. For the y plane picture they are very close to ellipses, for the x plane the trajectories have a somewhat

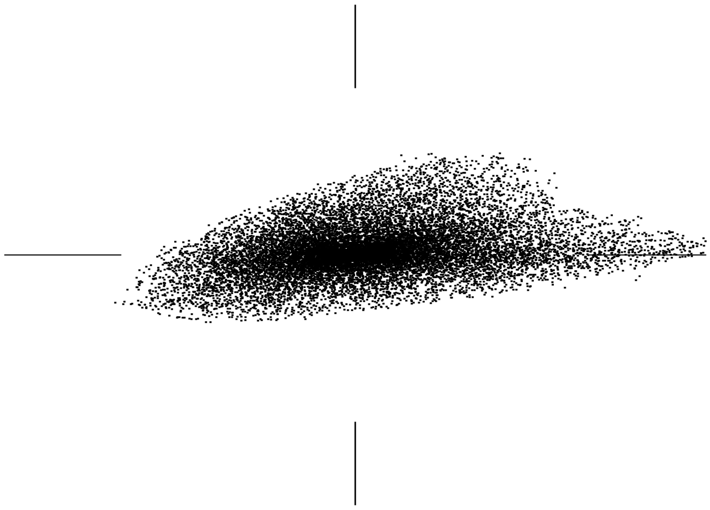

Fig. 2. x-plane phase portrait before the optimization with 85% skew quadrupole errors in dipoles.

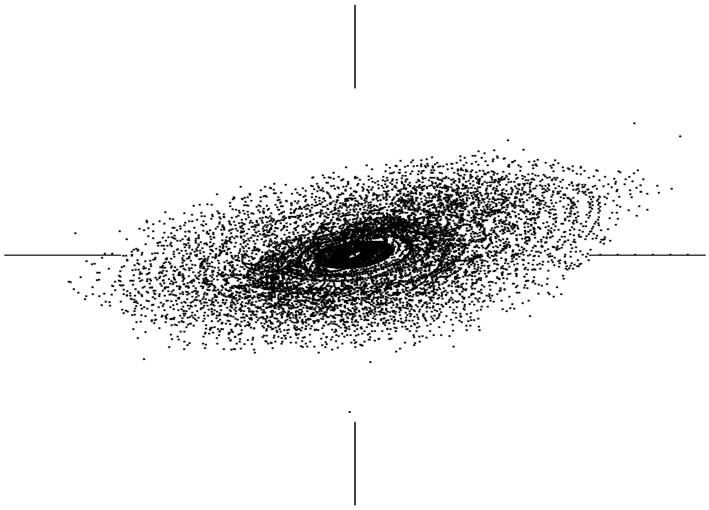

Fig. 3. y-plane phase portrait before the optimization with 85% skew quadrupole errors in dipoles.

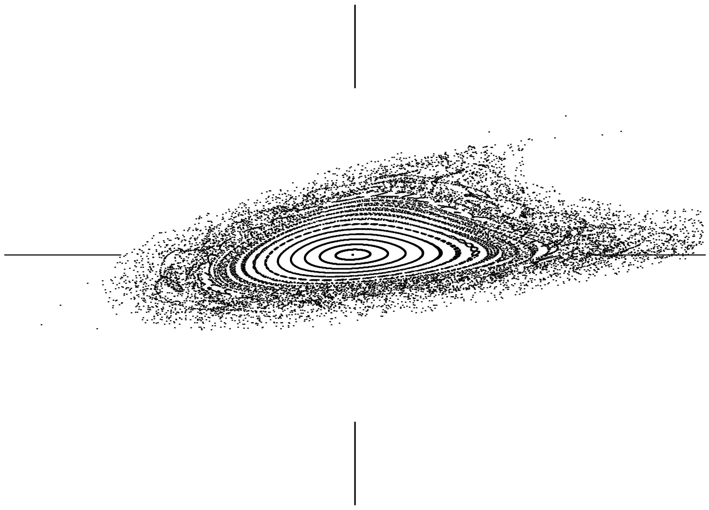

Fig. 4. x-plane phase portrait after the optimization with 50% skew quadrupole errors in dipoles, errors are fixed around each defocusing quadrupole.

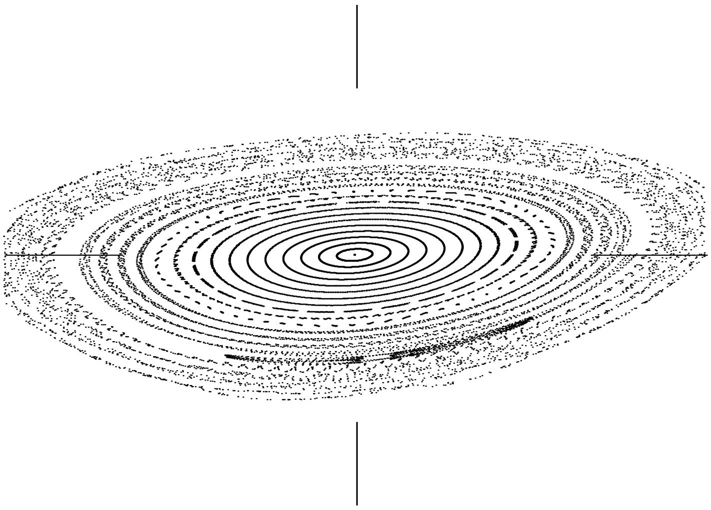

Fig. 5. y-plane phase portrait after the optimization with 50% skew quadrupole errors in dipoles, errors are fixed around each defocusing quadrupole.

triangular shape because of the proximity of a resonance. For unstable particles, the trajectories are fuzzy. For some extremely bad cases (like the one in the Figs. 2 and 3), the particles do not show stable behavior at all. After several turns most of the particles can be considered to be lost (their traces go beyond the scale of the picture).

The figures show great improvement in the behavior of the particles: fewer particles are lost, and the motion remains stable further from the reference particle that goes through the centers of all the magnets undisturbed. The picture of the x plane is still not perfect; there appears to be a possibility that the stable region can be increased further.

FODO 1	D*2	FQ	SQC RMV	D*2	D*2	DQ	D*2
FODO 2	D*2 FIX	FQ		D*2 FIX	D*2 FIX	DQ	D*2 FIX
FODO 3	D*2	FQ	SQC RMV	D*2	D*2	DQ	D*2
FODO 4	D*2 FIX	FQ		D*2 FIX	D*2 FIX	DQ	D*2 FIX
FODO 5	D*2	FQ	SQC	D*2	D*2	DQ	D*2
FODO 6	D*2 FIX	FQ		D*2 FIX	D*2 FIX	DQ	D*2 FIX
FODO 7	D*2	FQ	SQC	D*2	D*2	DQ	D*2
FODO 8	D*2 FIX	FQ		D*2 FIX	D*2 FIX	DQ	D*2 FIX
FODO 9	D*2	FQ	SQC	D*2	D*2	DQ	D*2
FODO 10	D*2 FIX	FQ		D*2 FIX	D*2 FIX	DQ	D*2 FIX
FODO 11	D*2	FQ	SQC	D*2	D*2	DQ	D*2
FODO 12	D*2 FIX	FQ		D*2 FIX	D*2 FIX	DQ	D*2 FIX
FODO 13	D*2	FQ	SQC	D*2	D*2	DQ	D*2
FODO 14	D*2 FIX	FQ		D*2 FIX	D*2 FIX	DQ	D*2 FIX
FODO 15	D*2	FQ	SQC RMV	D*2	D*2	DQ	D*2

Fig. 6. Correction scheme II, skew quadrupole errors in dipoles are fixed in each even FODO cell.

Scheme II (Fig. 6) answers this question. There are two differences in the second scheme: the dipoles are to be fixed in all the even FODO cells, while skew quadrupole correctors are located in odd cells. In addition, some of the correctors can be removed, and that makes the results even better (marked SQC RMV in Fig. 6).

Figures 7 and 8 present the results of particle tracking for correction scheme II. Clearly the improvement can be seen with the naked eye. All the particles remain stable for $10,000$ turns. This result is achieved with only one corrector strength per arc, which will work fine with one power supply for all the skew quadrupole correctors in each arc. Moreover, optimization gives only slightly worse results for the case where all the skew quadrupole correctors have the same strength around the entire ring, which means the correction scheme under consideration appears to be the best choice to implement.

6 Transfer Map Comparison

Since one of the aims of the optimization was the removal of coupling between the x and y planes, it is worth showing the first order transfer map of the machine before

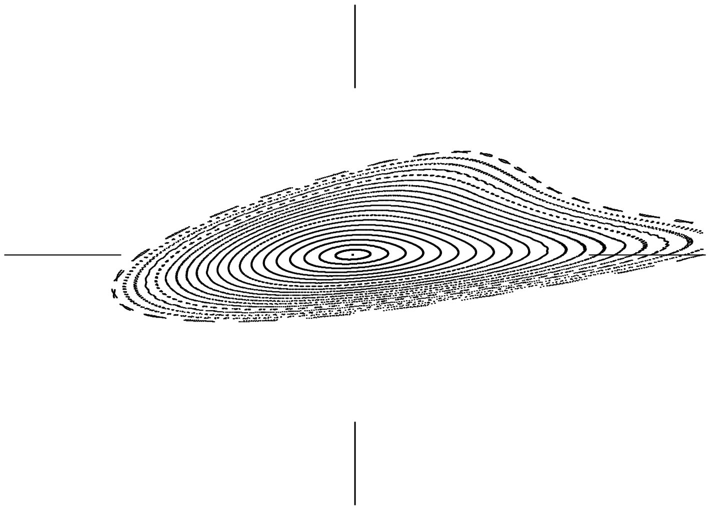

Fig. 7. x-plane phase portrait after the optimization with 50% skew quadrupole errors in dipoles, errors are fixed in each even FODO cell.

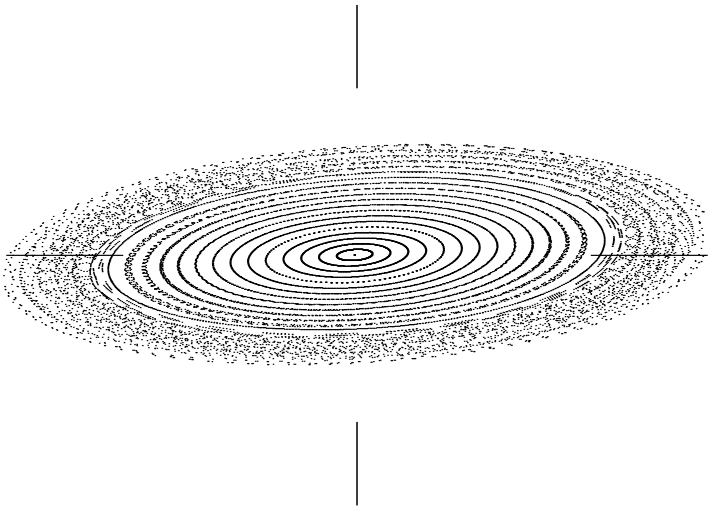

Fig. 8. y-plane phase portrait after the optimization with 50% skew quadrupole errors in dipoles, errors are fixed in each even FODO cell.

and after optimization. The map before optimization looks like this:

$$\begin{pmatrix} -0.7553149 & 0.3637584 & 0.0663166 & -0.0971958 \\ -0.8640196 & -0.9507699 & -0.4305946 & 0.1421480 \\ -0.0251393 & -0.0621722 & -0.8060247 & 0.3353297 \\ -0.4966847 & 0.0614653 & -0.7083212 & -0.9862029 \end{pmatrix}, \tag{4}$$

and after optimization:

$$\begin{pmatrix} -0.8023857 & 0.3107970 & \mathbf{0.0059011} & \mathbf{-0.00254055} \\ -0.7143330 & -0.9696470 & \mathbf{-0.0058075} & \mathbf{-0.00487758} \\ \mathbf{0.0043365} & \mathbf{-0.0022290} & -0.8445417 & 0.28890307 \\ \mathbf{-0.0077313} & \mathbf{-0.0060658} & -0.5409385 & -0.99907987 \end{pmatrix}. \tag{5}$$

The coupling terms shown in bold (four terms in the upper-right and lower-left corners) in (5) became up to 74 times smaller than in (4).

7 Conclusions

Without the use of AD techniques implemented in COSY INFINITY, achieving the results would be a hard, if not impossible task. The speed COSY tracking particles is remarkable. One procedure tracking in both x and y planes takes about 3 minutes for 7th order calculation or up to 8 hours (depending on the one-turn map) for order 11 on a 1.5 GHz Pentium processor.

The results of the study are very promising, and the correction scheme II presented above is the one being implemented during the Tevatron shutdown planned for Fall 2004.