doi:10.1017/S1431927615013045

A Comparison of Storage Ring Modeling with COSY INFINITY, ZGOUBI, and MAD8

Robert Hipple and Martin Berz

Department of Physics and Astronomy, Michigan State University, East Lansing, MI, USA

Currently there is significant interest in the use of storage rings to search for an electric dipole moment (EDM) in hadrons [1]. This requires utilizing the storage ring as a precision measuring device [2]. Part of understanding the detailed behavior of storage rings comes from careful analysis of fringe fields [3], but the various tracking codes available differ in their ability to model such behavior. It is the purpose of this paper to investigate these differences.

A major storage ring facility actively engaged in the search for hadron EDMs is the COoler SYnchrotron (COSY) at Forschungszentrum Jülich [4]. We modeled a simplified version of this storage ring using three well-known simulation codes – MAD8 [5], ZGOUBI [6] and COSY INFINITY [7]. MAD8 is a "transfer map" code of order 2, which means that the state of the particle in phase space is maintained as a vector, and the differential equations governing the motion of particle through the storage ring elements are represented by transfer maps. To track a particle through a system, one merely needs to perform map composition. MAD8 also has the capability to track particles symplectically using generating functions of third order [8].

ZGOUBI does not use the transfer map technique, but rather integrates the Lorentz equation by time stepping based on a Taylor series in path length. The coefficients of the Taylor series in time are determined by an additional Taylor expansion of the magnetic field, to fifth order maximum, if the fields are given analytically, and by an out of plane expansion based on numerical differentiation otherwise. ZGOUBI has few programming capabilities beyond a simple looping mechanism to provide multiple passes through an optical system. ZGOUBI provides support for fringe fields via Enge coefficients [9]. The software distribution also includes a powerful post processing module called ZPOP for plotting and data visualization.

COSY INFINITY is a combination of the advantages of the transfer map approach and integration codes. It is primarily a transfer map code, but utilizes integration internally to create highly accurate maps for fringe fields [10]. Built into COSY INFINITY is an interpreter for the specialized COSYScript programming language [11], which allows the researcher to simulate charged particle optics systems to a high degree of accuracy using the techniques of Differential Algebra. Fringe fields are specified by Enge coefficients which can be input by the user to model actual field measurements, or a default set of typical values can be chosen.

To establish a baseline, we begin with a simplified hard-edge model of the COSY storage ring (Figure 1). The ring is highly symmetric, incorporating two 40 m telescope regions, and two 52 m arcs. Each arc is composed of three identical bending segments, each with mirror symmetry. The bending elements are rectangular dipoles. There are 16 sextupoles (not indicated on the figure) at various locations around the lattice. Our model incorporates only the bending and focusing elements – the sextupoles in the actual lattice are not modeled. After implementing the storage ring elements into the three codes, we confirm that the first order transfer matrices are essentially identical (Table 1).

COSY INFINITY

-0.9774877	-1.078548	0.00	0.00
0.03521565	-0.9841743	0.00	0.00
0.00	0.00	-0.5176308	-10.90340
0.00	0.00	0.06520659	-0.5583641

MAD8

-0.9774876	-1.0785556	0.00	0.00
0.03521571	-0.984175	0.00	0.00
0.00	0.00	-0.5176308	-10.90340
0.00	0.00	0.06520659	-0.5583641

ZGOUBI

-0.9774910	-1.0783900	0.00	0.00
0.03521423	-0.984180	0.00	0.00
0.00	0.00	-0.5176260	-10.90340
0.00	0.00	.06520679	-0.5583590

Table 1. First order transfer matrices for the three codes without fringe fields. Note that COSY INFINITY and MAD8 use transfer matrices and thus naturally agree to high accuracy, whereas ZGOUBI calculates the transfer map as a result of integration of nearby orbits which is slightly less accurate.

Having verified that our lattices agree to first order, we perform some initial tracking runs and compare the results. Figure 2 shows the tracking pictures for a single 970 MeV/c proton over 2000 turns with cosine-like initial conditions at transverse amplitude < 1 cm. There is quite good agreement between MAD8 and COSY INFINITY when performing tracking at MAD8's maximum order of 2, including the inward spiral characteristic of non-symplectic tracking. The output for ZGOUBI also shows a small violation of symplecticity, but apparently ZGOUBI's accuracy is higher than that of second order transfer maps.

With COSY INFINITY and MAD8 we can enable symplectic tracking [12], which ZGOUBI does not

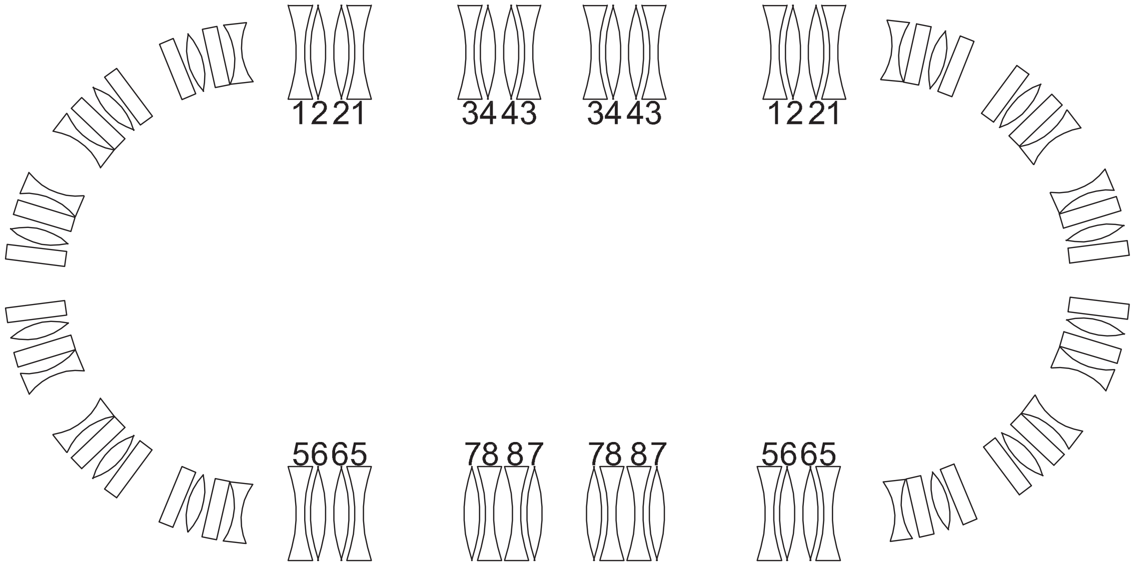

Figure 1. Simplified model of the COSY storage ring at Forschungszentrum Jülich.

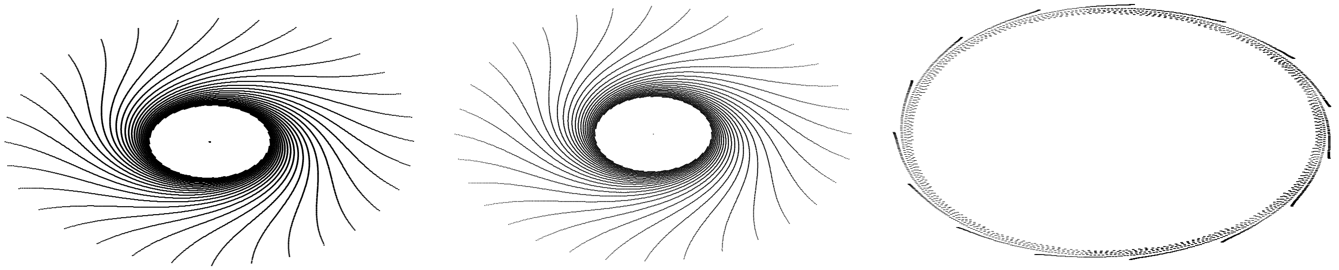

Figure 2. Second order tracking in the horizontal plane created by COSY INFINITY (left) and MAD8 (center), compared with ZGOUBI (right) which does not use transfer maps, under identical initial conditions and lattice parameters. No symplectification is used, showing that second order is insufficient to describe the dynamics.

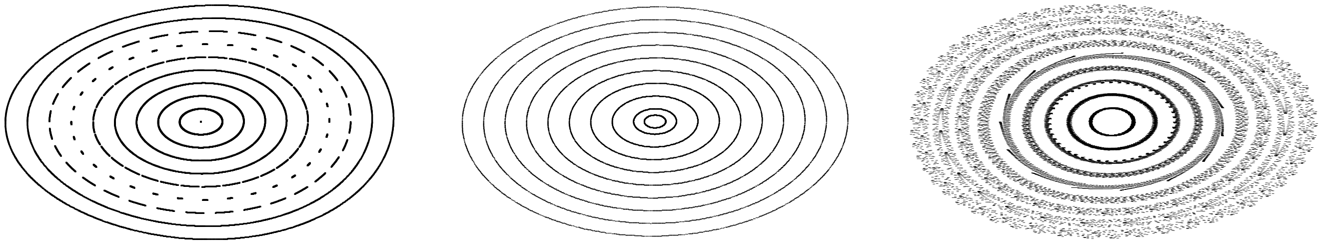

Figure 3. Varying transverse initial conditions from 1 cm to 9 cm in the horizontal plane for second order tracking with COSY INFINITY (left) and MAD8 (center) and compared with ZGOUBI (right). Symplectic tracking is enabled in COSY INFINITY and MAD8. This is not available in ZGOUBI, which shows a widening of the orbit bands indicative of violation of symplecticity.

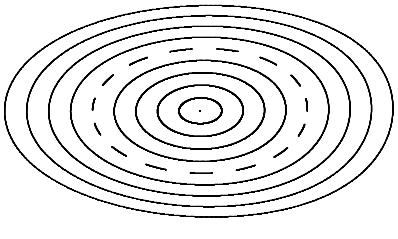

Figure 4. Same tracking as Figure 3, now utilizing COSY INFINITY's 9th order tracking without symplectification, still without fringe fields. The high order of the map and resulting accuracy avoids the violation of symplecticity visible in Figure 2. Even to this high order, there is very little nonlinearity evident in the dynamics.

support. Figure 3 exhibits tracking pictures with symplectic tracking enabled. They are virtually identical for COSY INFINITY and MAD8, the phase space orbits having a well-defined elliptical shape. This establishes a baseline between COSY INFINITY, ZGOUBI and MAD8. Turning to the question of dynamic aperture, we track particle orbits in 1 cm steps from the reference orbit out to 9 cm – twice the physical aperture of the actual ring. We see little significant deviation from linear behavior across all three codes. So far, with COSY INFINITY and MAD8, we are still tracking with second order transfer maps. With COSY INFINITY we can push to higher orders with very little extra processing time. Figure 4 is the same

-0.9739104	1.954368	0.00	0.00
0.01832738	**-1.063567**	0.00	0.00
0.00	0.00	-0.7993542	-6.705731
0.00	0.00	0.05219644	-0.8131372

COSY INFINITY

-0.9618310	2.08049	0.00	0.00
.02259699	**-1.089850**	0.00	0.00
0.00	0.00	-0.209283	17.4232
0.00	0.00	-.05576599	-.13560600

ZGOUBI

Table 2. First-order transfer matrices for COSY INFINITY and ZGOUBI with full fringe fields enabled. The horizontal trace (the sum of the bold matrix elements) is greater than 2 in magnitude, which indicates instability.

tracking run, this time to 9th order in the transfer maps. Still there is very little nonlinearity evident in the dynamics, and the dynamic aperture appears unlimited.

Judging from these results alone, one might conclude that the lattice is remarkably stable. We now investigate the effect of incorporating fringe fields. COSY INFINITY has the convenience of enabling a default set of fringe field profiles with the single command "FR 3". Table 2 shows the COSY INFINITY first order transfer map with fringe fields turned on. Notice that the motion in the horizontal direction is now *unstable* ($|\mathrm{Trace}| > 2$).

We can confirm this by adding identical fringe fields to ZGOUBI and checking the computed transfer matrix (Table 2). A similar horizontal instability is indicated in the ZGOUBI transfer matrix as well. It is apparent that some adjustment of the magnet strengths is necessary to stabilize the beam. Without making any specific assumptions regarding which adjustments to make, we choose an arbitrary, minimally invasive approach which respects the existing symmetries of the lattice. The numbered elements in Figure 1 indicate quadrupoles which are equal in field strength. By preserving this equality, we maintain the symmetry that was designed into the lattice. This allows 8 degrees of freedom. Since we are trying to match 8 elements of the transfer matrix, we require only six degrees of freedom due to the unity determinant. By utilizing an implementation of the MINPACK LMDIF optimizer built into COSY INFINITY [13], we find a (not necessarily unique) set of quadrupole strength multipliers which achieve our objective:

$$\lambda_1 = 1.022183, \quad \lambda_2 = 1.019331, \quad \lambda_3 = 1.009453, \quad \lambda_4 = 1.019948,$$
$$\lambda_5 = 0.999410, \quad \lambda_6 = 1.019427, \quad \lambda_7 = 1.031603, \quad \lambda_8 = 1.013424.$$

We see this requires an adjustment of only a few percent in the quadrupole strengths. Implementing these changes into all three codes, we successfully restore the lattice to its original design transfer matrix.

Figure 5 shows the resulting tracking pictures for COSY INFINITY. This run was 9th order in the transfer matrices, with the fringe fields enabled for all lattice elements. In the horizontal direction, nonlinearities are readily apparent. The shape of the phase space ellipses are seen to vary with increasing distance from the reference orbit. At 8 cm, seven "islands" of stability appear. In the vertical direction, there is substantial beam loss at distances > 6 cm. Neither of these phenomena are visible in the hard-edge approximation.

Figure 6 shows the same tracking experiment using ZGOUBI. It is interesting to note that the seven

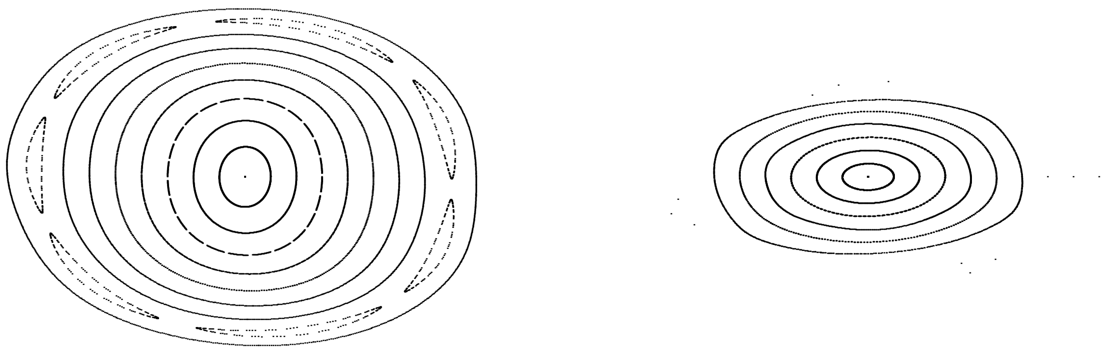

Figure 5. COSY INFINITY 9th order tracking with full fringe field simulation capabilities in vertical (left) and horizontal (right) planes.

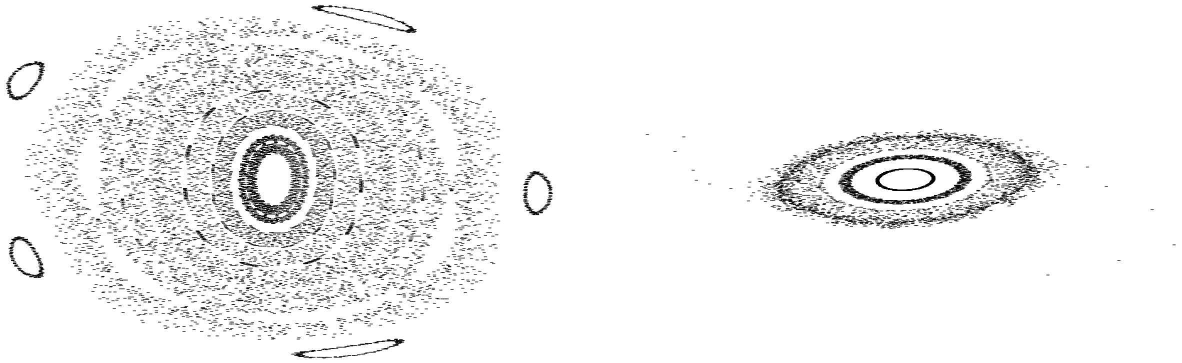

Figure 6. ZGOUBI tracking with full fringe field simulation capabilities in vertical (left) and horizontal (right) planes. The fringe field Enge coefficients are identical to those used in COSY INFINITY. The rough structure and stability boundaries are similar to those of Figure 5, but symplecticity violations are apparent.

"islands" revealed by COSY INFINITY are visible with ZGOUBI as well but these have the appearance of "gaps" in the tracking picture. In the vertical direction, ZGOUBI also agrees with COSY INFINITY's prediction of substantial beam loss at distances greater than 6 cm.

In conclusion, modeling a storage ring without taking fringe fields into account provides an overly optimistic dynamic aperture and an incomplete picture of the dynamics. Although COSY INFINITY and ZGOUBI differ in their methods of tracking, both codes hint at similar nonlinear dynamics and agree very well for the linear motion with fringe fields. This agreement across codes, despite completely different tracking methods, gives confidence that the predictions made reflect real physics. There is a substantial difference in run time, however (Table 3), and for moderately large numbers of turns and particles, COSY INFINITY is about three orders of magnitude faster than the identical simulation in ZGOUBI. MAD8, which does a good job of modeling within the SCOFF (Sharp Cutoff Fringing Field) approximation, has limited fringe field capabilities. This limits its usefulness for lower momentum tracking over many turns, precisely the type of tracking required for mid-size storage rings [14].

Number of Turns	CPU time (seconds)	
	COSY INFINITY	ZGOUBI
10^3	25.082	183.78
10^4	25.297	1831.4
10^5	27.132	18717
10^6	45.343	
10^7	228.26	
10^8	2049.4	
10^9	20193	

Table 3. Comparison of tracking execution times of COSY INFINITY and ZGOUBI at their respective maximum precisions (order 9 transfer map for COSY INFINITY, 5th order Taylor series integration for ZGOUBI). ZGOUBI execution times are proportional to the number of turns and around 0.187 seconds per turn per particle. COSY INFINITY requires an initial investment in the computation of a transfer map, but for larger turn numbers tracks for 1/50000 of a second per turn.

References:

[1] A Lehrach *et al*, XIV Advanced Research Workshop on High Energy Spin Physics (2012) p. 287.
[2] A Lehrach, private communication.
[3] H Wollnik, “Optics of charged particles”, (Academic Press, Orlando, 1987).
[4] U Bechstedt *et al*, Nuclear instruments and methods in physics research B **113** (1996), p. 26.
[5] H Grote and C Iselin, “The MAD program (methodical accelerator design), version 8.13/8, user’s reference manual”, CERN/SL/90-13 (AP) (Rev. 4), (CERN, Geneva, 2012).
[6] F Méot, “ZGOUBI users’ guide”, C-AD/AP/470, (Brookhaven National Laboratory, Upton, 2013).
[7] M Berz and K Makino, “COSY INFINITY 9.1 beam physics manual”, MSUHEP 060804-rev, (Michigan State University, East Lansing, 2013), http://cosyinfinity.org.
[8] C Iselin, “The MAD program (methodical accelerator design), version 8.13, physical methods manual”, CERN/SL/92 (AP), (CERN, Geneva, 2012).
[9] H Enge in “Focusing of charged particles”, ed. A Septier, (Academic Press , New York, 1967), p. 203.
[10] M Berz, “Modern map methods in particle beam physics”, (Academic Press, San Diego, 1999).
[11] M Berz and K Makino, “COSY INFINITY 9.1 programmer’s manual”, MSUHEP 101214, (Michigan State University, East Lansing, 2011), http://cosyinfinity.org.
[12] S Manikonda, M Berz and B Erdélyi, Institute of Physics CS **175** (2004), p. 299.
[13] JJ Moré, BS Garbow and KE Hillstrom, “User guide for MINPACK-1”, ANL-80-74, (Argonne National Laboratory, Argonne, 1980).

[14] We are grateful for financial support from the US Department of Energy under grant DE-FG02-08ER41546. We thank Kyoko Makino at Michigan State University for helpful comments and suggestions, and Denis Zyuzin and Marcel Rosenthal at Forschungszentrum Jülich for their assistance in procuring the lattice model.