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COSY INFINITY version 8

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Abstract

The latest version of the particle optics code COSY INFINITY is presented. Using Differential Algebraic (DA) methods, the code allows the computation of aberrations of arbitrary field arrangements to in principle unlimited order. Besides providing a general overview of the code, several recent techniques developed for specific applications are highlighted. These include new features for the direct utilization of detailed measured fields as well as rigorous treatment of remainder bounds. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Code; Differential algebraic method; Computation

1. The Code COSY

COSY INFINITY [1] is a code for the simulation, analysis and design of particle optical systems, based on differential algebraic (DA) methods [2–4]. Currently there are a total of about 270 registered users.

The code has its own scripting language with a very simple syntax [5]. For the utilization of DA tools, the code is object oriented, and it allows dynamic adjustment of types. The engine for DA operations [6,7] is highly optimized for speed and fully supports sparsity, which greatly enhances performance for systems with midplane symmetry. There are also conversion tools to transform any lattice in standard MAD input or in the Standard eXchange Format (SXF format) to a program in COSY language. The compiled code can either be executed directly or saved in a binary file for inclusion in a later code.

The compiler has a rigorous syntax and error analysis and is comparable in speed to compilers of other languages. The object oriented features of the code are not only useful for the direct use of the differential algebraic operations, but also for other important data types including intervals and the new type of remainder-enhanced differential algebras.

2. Simultaneous integration of reference orbit and map

Besides very special cases of simple elements, the computation of a transfer map requires numerical integration. In Refs. [2,8] it is shown how maps of any order can be obtained for arbitrary fields, based on mere integration of suitable DA objects.

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COSY uses a Runge Kutta integrator of order eight with automatic step size control based on a seventh-order scheme for this purpose. However, the equations of motion of the map require the knowledge of the momentary curvature of the reference orbit, and under the presence of acceleration, the momentary energy of the reference orbit. For practical systems, these quantities themselves are usually obtained via numerical integration.

An important simplification of this approach was recently introduced into COSY in connection with the detailed study of high-order maps of accelerating cavities. Since the real number reference orbit motion and the DA transfer map motion are coupled, the equations of motion for both reference orbit and map were solved simultaneously as one global set of equations. In this framework, part of the differential equations are real, and part are DA. In practice, this necessity greatly benefits from the fact that COSY allows dynamic typing, i.e. the adjustment of data types at run time, within COSY's object oriented environment. In this way, the map integration becomes more stable and, for complicated accelerating structures, shows significant computational efficiency gains.

3. Standard fringe field calculation

From its earliest versions, COSY has featured various methods to account for fringe field effects in the calculation, including the choice of model functions to represent the fringe fields. The standardized model is based on the description of the s-dependence of multipole strengths by an Enge function

$$F(z) = \frac{1}{1 + \exp(a_1 + a_2\cdot(z/D) + \cdots + a_6\cdot(z/D)^5)}\,.$$

The pictures in Fig. 1 show the fringe field models adopted by default in COSY for dipoles and for quadrupoles. In both cases, the variable z measures the distance to the effective field boundary. It coincides with the arc length s along the reference trajectory in the case of multipoles, but in the case of dipoles it takes into account tilts and curvatures of the effective field boundary. D is the full aperture. Besides the default Enge functions, the user can load his own set of Enge coefficients a_1–a_6.

The numerical integrator described in the previous section computes the effects of analytically described fringe field exactly in the equations of motion. As long as an analytical expression of the fringe field is provided, COSY calculates even very detailed fringe-field effects, but the computational expense is prone to be higher than for main field maps.

COSY also provides other approaches for the computation of approximate fringe-field effects that are much less costly computationally. The first one uses approximate fringe fields with an accuracy comparable to the fringe-field integral method. The other one is the SYSCA method, which uses a combination of geometric scaling in TRANSPORT coordinates and symplectic rigidity scaling [9,10]. It uses parameter-dependent symplectic representations of fringe-field maps stored in files. These can either be produced by the user or taken from the COSY shipment. This method computes fringe fields with very high accuracy at very modest cost.

Another feature available from the early days of COSY is an element to compute the map of a general optical element characterized by the values of multipole strengths and reference curve and their derivatives supplied at points along the independent variable s. In principle, this element can be used for the calculation of any particle optical system. But in practice, it first requires the determination of the curvature as a function of s, which often requires numerical integration. Furthermore, it is necessary to provide high-order derivatives, which are frequently not readily available.

4. The azimuthally dependent sector magnet

While COSY has a large library of electromagnetic elements, sometimes it is necessary to allow for a more detailed description of the field. An important example is the precise analysis required for modern nuclear spectrographs. In such a case, a custom-made COSY element with an analytically described field model can help, but sometimes there is no other way than utilizing the measured field data in the computation, which has to be supplied

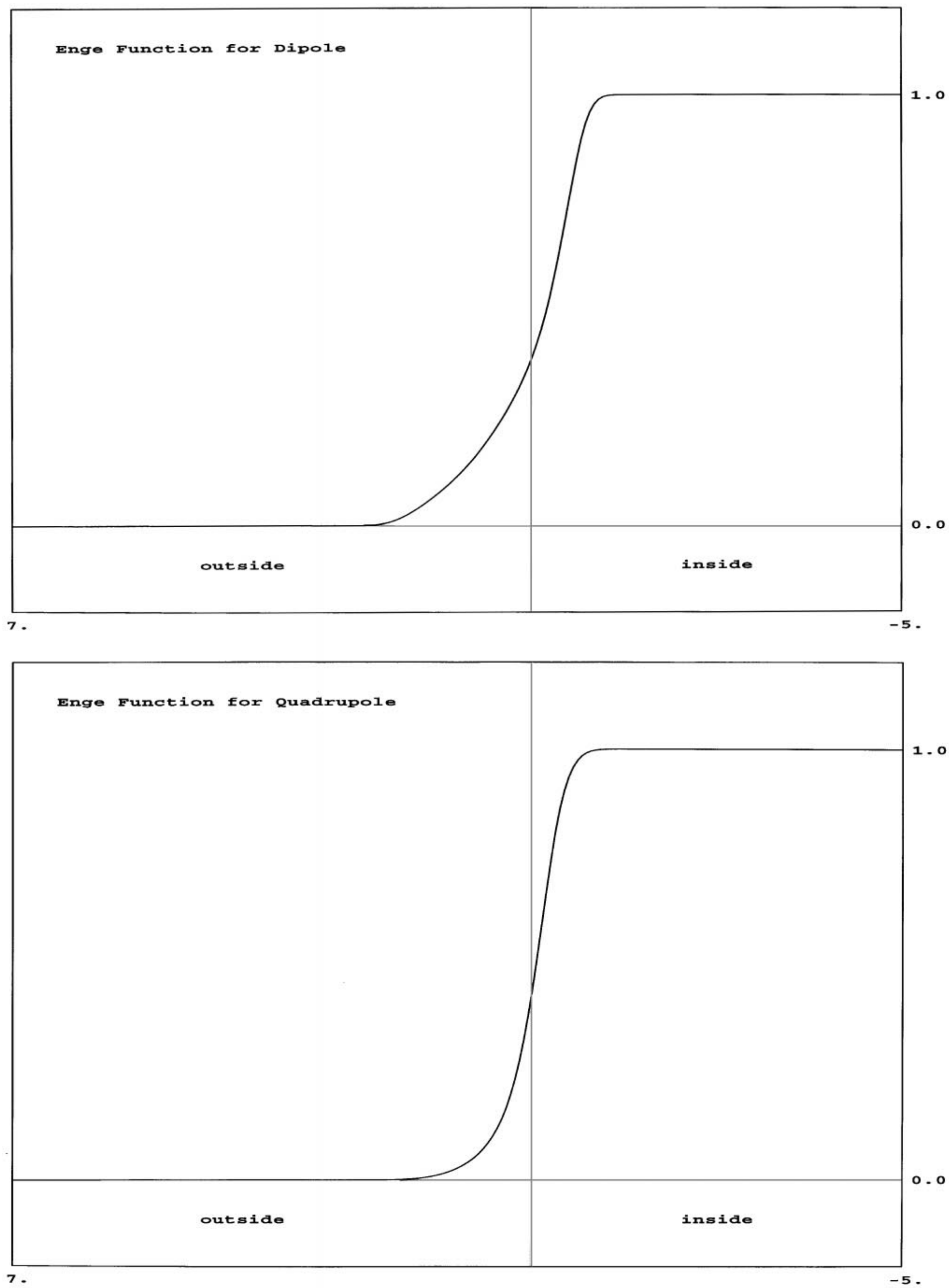

Fig. 1. Fringe field model by Enge function for dipoles (top) and quadrupoles (bottom) by default in COSY. The horizontal axis denotes z/D. Pictures are generated with COSY's graphics environment.

to the equations of motion in an appropriate way to be integrated by the DA integrator discussed earlier.

The methods we will discuss in this section are used extensively in the simulation of the S800 Spectrograph at the National Superconducting

Cyclotron Laboratory at Michigan State University [11], which uses the approach of high-order reconstructive correction [12] in COSY to achieve its high-energy resolution, as well as the various spectrographs at the other laboratories. The detailed field description of huge bending magnets is key to the precise analysis of such a system, and a rough estimate in the S800 case shows a need of a relative accuracy of 10^{-4} of the fields.

The conventional bending magnets in COSY are a homogeneous dipole with edge angles and curvatures at entrance and exit, an inhomogeneous bending magnet with the midplane radial field dependence given by

$$F(x) = F_0\left[1 - \sum_{i=1}^{5} n_i\left(\frac{x}{r_0}\right)^i\right]$$

where r_0 is the bending radius, and an inhomogeneous bending magnet with shaped entrance and exit edges. To this main field model, Enge-type fringe fields are tacked on. A new bending magnetic element in COSY allows to specify the two-dimensional structure of the main field in polar coordinates via

$$F(r,\phi) = \sum_{i=0}^{3}\sum_{j=0}^{3} A_{ij}(r-r_0)^i(\phi-\phi_0/2)^j$$

where ϕ_0 is the angle of deflection of the element. For the description of edge effects, Enge-type fringe field effects as well as the consideration of edge angles and curvatures at entrance and exit are included.

Another yet more comprehensive way to treat complicated bending magnets is based on direct specification of measured field data [5,13].

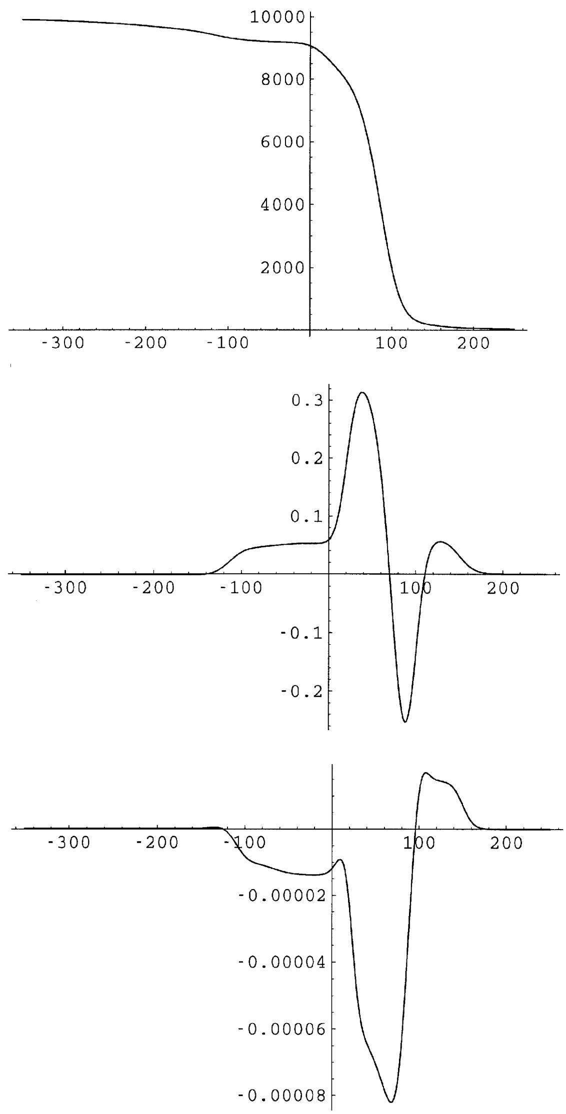

Fig. 2. The Quadrupole, Duodecapole, and 20 pole strengths in the fringe fields of the LHC High Gradient Quadrupoles.

5. The multipole based on tabulated data

In some instances it is not possible to rely on simple models for the description of the fringe fields of multipoles. An important case in point is the study of the High Gradient Quadrupoles of the interaction regions of the LHC. Fig. 2 shows the behavior of the quadrupole strength as well as the 12 and 20 pole strengths. Because of the complicated structure, fitting the quadrupole term merely with Enge functions is difficult, and clearly the higher order terms are not very amenable to detailed description by Enge functions.

For such purposes when there is no good analytical model available to describe the field, it is desirable to directly utilize measured field data for the computation of the map. Following the

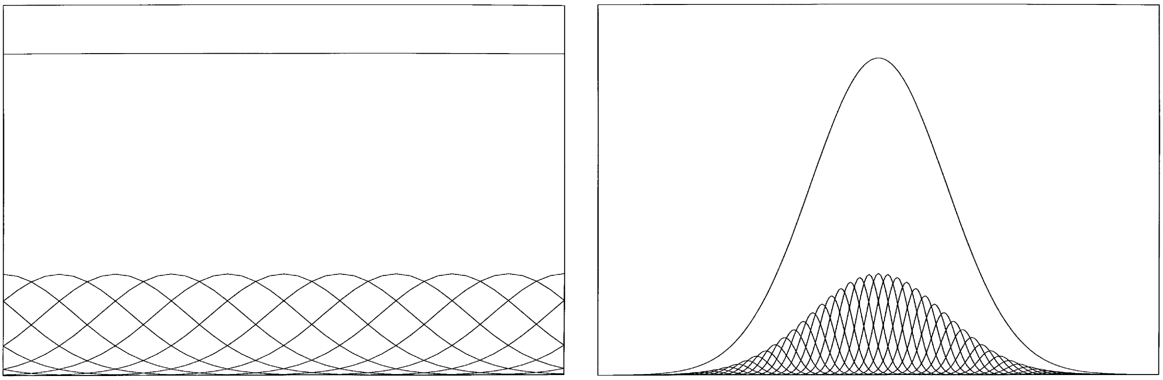

Fig. 3. Gaussian wavelet representation for $f(x) = 1$ (left) and $f(x) = \exp(-x^2)$ (right). Pictures are generated with COSY's graphics environment.

conventional DA integration scheme to obtain maps to arbitrary order [8], it is necessary to know both the multipole strengths as well as their higher order derivatives. Thus, an interpolation based on measured multipole terms has to assure differentiability. The Gaussian wavelet representation

$$F(x) = \sum_{i=1}^{N} A_i \frac{1}{\sqrt{\pi}S} \exp\left[-\frac{(x - x_i)^2}{\Delta x^2 S^2} \right] \tag{1}$$

has proven very well suited for this purpose, while at the same time providing localization and adjustable smoothing of the data. In Eq. (1), A_i are the values of data at N equidistant points x_i spaced by the distance Δx, and S is the control factor of the width of Gaussian wavelets. Pictures in Fig. 3 show the Gaussian interpolation of one dimensional functions as a sum of Gaussian wavelets for a constant function $f(x) = 1$ and a non-constant function, as an example, a Gaussian function $f(x) = \exp(-x^2)$.

The method can also be extended to allow for a two-dimensional description of measured field data in the midplane that is often available for high-quality bending magnets like those of the S800 [5,13]. The time consuming summation over all the Gaussians, especially in two-dimensional case, can take an advantage of the quick fall-off of the Gaussian function, hence the summation of only the neighboring Gaussians is enough for the accuracy yet greatly improves the computational efficiency.

6. Remainder-enhanced differential algebraic method and other features

The highlight of version 8 of COSY from the perspective of computational mathematics is a new technique, the remainder-enhanced differential algebraic (RDA) method, which computes rigorous bounds for the remainder terms of the Taylor expansions along with the Taylor polynomials. The details of the method are found in Refs. [14–17]. For beam physics, it opens the capability of the determination of rigorous bounds for the remainder term of Taylor maps [18], and it can estimate guaranteed stability times in circular accelerators combined with methods to determine approximate invariants of the motion [19,20].

Other features in COSY include methods for symplectic tracking [2,21], normal forms [2,22], tools used for the design of fifth-order achromats [23,24], and the analysis of spin motion [25,26], which has gained importance connected to the desire to accelerate polarized beams. There are also various technical tools including a new interactive graphics based on PGPLOT. The demo file of the code, which is a part of the COSY shipment, provides a good overview over the key features in beam physics.

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