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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

The influence of fringe fields on particle dynamics in the Large Hadron Collider

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Abstract

The need of maximizing luminosity in the Large Hadron Collider requires the use of high-gradient quadrupoles in the interaction region. These quadrupoles combine relatively short length, large aperture, and short focal length with a rather peculiar configuration of the return coils, all of which enhances the relevance of their fringe field effects. The influence of resulting nonlinearities on the dynamics is analyzed via high-order maps determined with differential algebraic (DA) techniques and the code COSY INFINITY. Normal form methods are utilized to determine amplitude-dependent tune shifts as well as resonance strengths. An analysis based on a detailed description of the fringe field of the superconducting quadrupoles reveals that the strength of resonances increases by more than one order of magnitude, and that amplitude-dependent tune shifts are enhanced substantially.

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1. Introduction

Beam physical systems can be described by a map $\mathscr{M}$ that relates coordinates z_i in an initial plane to coordinates z_f in a final plane via

$$z_f = \mathscr{M}(z_i). \tag{1}$$

All information about the optical system is contained in this map. Since the relationship between initial and final coordinates is weakly nonlinear, the map is often represented by its Taylor expansion. We use the differential algebraic method implemented in the code COSY INFINITY [1,2] to obtain a high-order expansion of the map. From this map we can calculate the amplitude-dependent tune shifts of the system using the normal form algorithm described in Ref. [3].

Accelerator lattices are usually described by the position, length and field strength in the main body

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of their elements. The field of the magnets is considered to change from zero to the value in the main field at the magnet entrance and drop again to zero at the magnet exit. Although this approximation is frequently employed in beam physics, it is rather unrealistic and at least for the short-term behavior leads to considerable effects. Using COSY INFINITY it is possible to take into account the effect of the exact shape of the magnetic field at the ends of the magnet. To this end the magnet is split into a main section in which the field is independent of the particle optical coordinate s, and an s-dependent element representing the fringe field. The fringe field map, which has finite length, is composed of two negative drifts, to produce a zero-length insertion [4].

Besides allowing for the computation of the amplitude-dependent tune shifts, the DA Normal Form algorithm implemented in COSY INFINITY further allows to calculate the resonance strengths, which tell how sensitive the system is to certain resonances. For details see Ref. [3]. The code COSY INFINITY is available from [5].

2. Magnetic field data

A general form of the potential in cylindrical coordinates can be written as Taylor expansion with respect to the radius and Fourier expansion with respect to the angle via

$$V = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} M_{k,l}(s) \cos(l\phi + \theta_{k,l}) r^k. \tag{2}$$

Inserting this expansion into the Laplacian yields the recursion relation

$$M_{l+2n,l}(s) = \frac{M_{l,l}^{(2n)}(s)}{\prod_{v=1}^{n} (l^2 - (l+2v)^2)}. \tag{3}$$

We are studying the quadrupole component $M_{2,2}(s)$, which can be calculated from the Fourier components of the field in the end region given in Ref. [6]. Additional data for the quadrupole component at different radii r were available, allowing to verify that the off-axis expansion given by Eqs. (2) and (3) agrees with the measured data.

We model the quadrupole strength $M_{2,2}(s)$ data using an Enge function with six parameters of the form

$$F(s) = \frac{1}{1 + \exp(a_1 + a_2(s/d) + a_3(s/d)^2 + \cdots + a_6(s/d)^5)}, \tag{4}$$

where s is the Cartesian distance to the field boundary. The quantity d is the full aperture, respectively, twice the radius, of the quadrupole. The nonlinear optimizers implemented in COSY INFINITY were used to adjust the Enge coefficients a_1–a_6 such that the resulting function agrees with the data. In the optimization process we shift $F(s)$ such that the effective field boundary coincides with the origin.

The results are that the Enge function fits the data for return and lead end very well, showing that this model class is sufficient even for the rather peculiar shape of the fringe field and that the given data at different radii agrees with the result calculated from Eqs. (2) and (3) using $F(s)$.

3. Lattice description

In our analysis, we use the LHC lattice model Version 5.0, which is available from Ref. [7]. We have written a tool [5] to convert the lattice description given in the @-output format of MAD 8.0 to COSY language.

In the present study the ring is subdivided in three regions, the two inner triplets (see Fig. 1) left and right of the interaction point 5, for which the detailed field data described above is available, and the rest of the ring. We calculate the linear map for the rest of the ring, and the seventh-order map, with and without fringe fields, for the triplets. Using this approach we can study the nonlinear effects of the fringe fields in the triplets, which is the most critical part of the lattice, as explained in the abstract.

4. Results

We computed the amplitude-dependent tune shifts with and without fringe fields. The result for

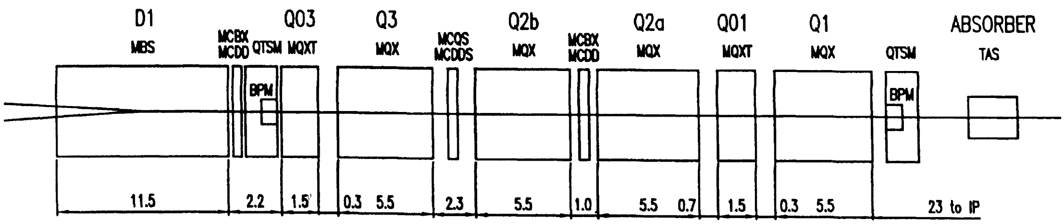

Fig. 1. Layout of the left low-β triplet including D1.

Table 1
Amplitude-dependent tune shifts. Shown are the dependence on powers of the horizontal (first column) and vertical (third column) emittance of the beam

Without fringe fields	With fringe fields		Order	Exponents			
	Original	Center tune refitted					
0.310000	0.303921	0.310000	0	0	0	0	0
58.5018	592.429	610.796	2	2	0	0	0
20.45303	873.057	902.598	2	0	0	2	0
8352.403	1889188	2191916	4	4	0	0	0
− 3360.344	− 13279249	− 13536156	4	2	0	2	0
9216.0682	11570624	12740611	4	0	0	4	0
13692934	− 399181479286	− 422995493944	6	6	0	0	0
− 51336671	88048290859	96863534946	6	4	0	2	0
124626604	− 1035625378625	− 1159771282208	6	2	0	4	0
− 15542851	1394274127638	1515548018372	6	0	0	6	0

the horizontal $x - a$ plane is given in Table 1. The table shows two effects. First, the nonlinear tune shifts, shown in their polynomial expansion in terms of emittance, increase considerably. Second, the center tune decreases by 6×10^{-3}. This can be corrected using the trim quadrupoles in the LHC lattice. But even after refitting the center tune to its original value, the nonlinear tune shifts are considerably bigger with fringe fields than without fringe fields. In fact they are still of the same order of magnitude as without refitting the tune. The tune footprint shown in Figs. 2 and 3 clearly demonstrates the change in the dynamics of the system.

Compared to the previous study done by Méot et al. [8], the center tune change shown here is larger. This is most likely attributable to the fact that the two studies use different sets of Enge coefficients, and the ones utilized here are in fact much more extended and irregularly shaped than conventional fringe fields. On the other hand, both studies show a similar tune shift with amplitude.

In order to further investigate the changes in the dynamical behavior of the system we study the resonance strength [3,9]. To compare the resonance strengths with and without fringe field, we calculate the average absolute value of the resonance strengths for every order. The result is given in Fig. 4. It shows clearly that the resonance strength increases by at least one order of magnitude on average, being even stronger for higher orders.

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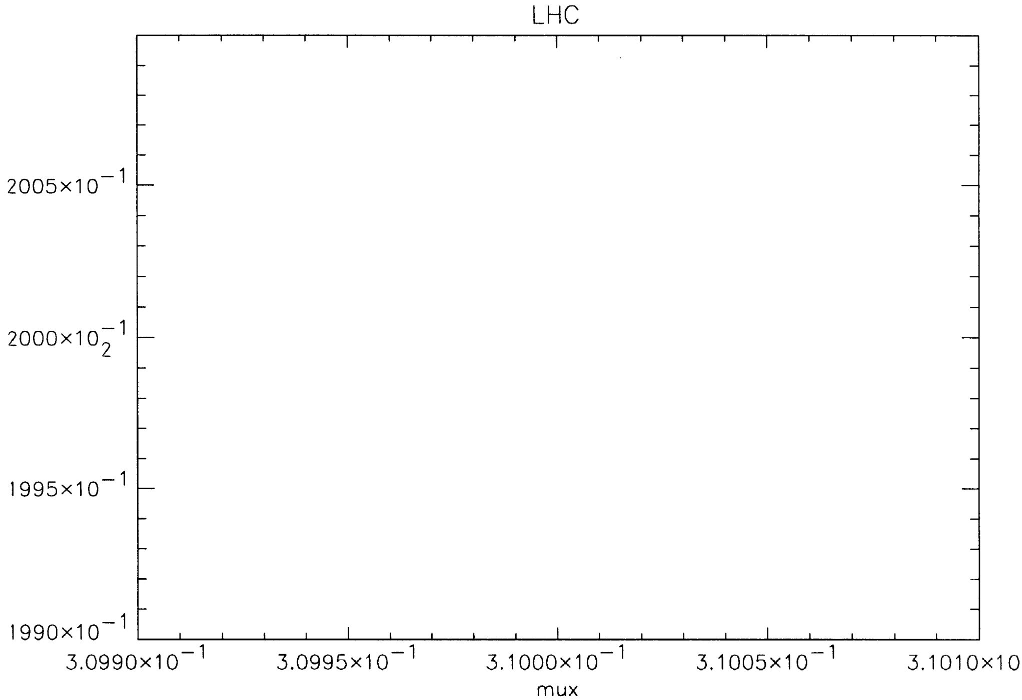

Fig. 2. Tune footprint without fringe fields.

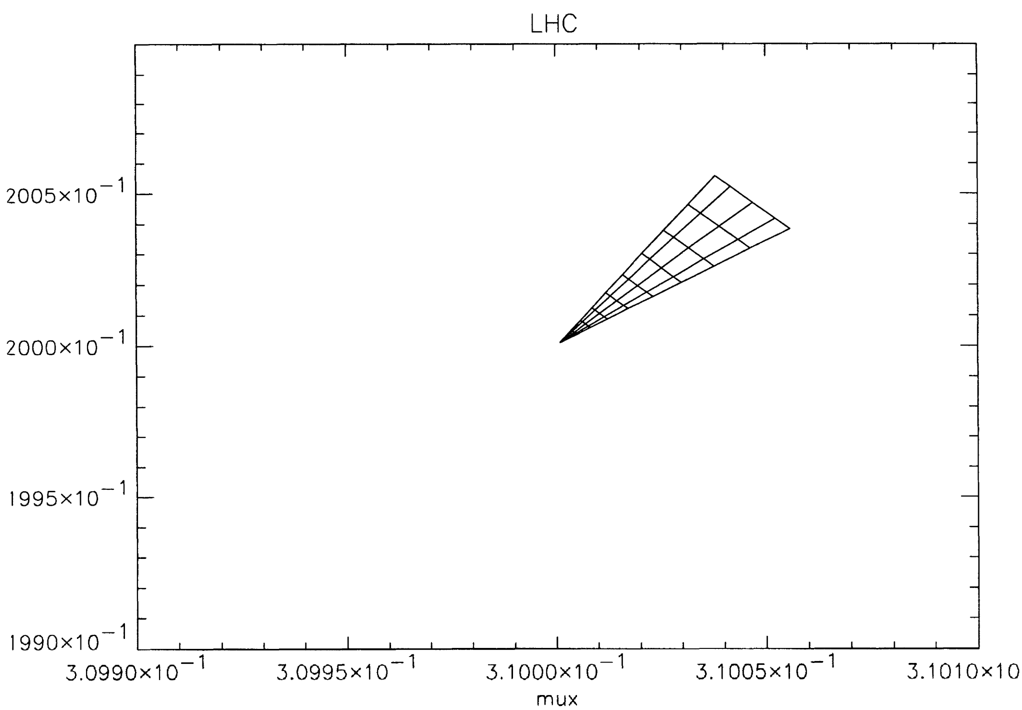

Fig. 3. Tune footprint with fringe fields.

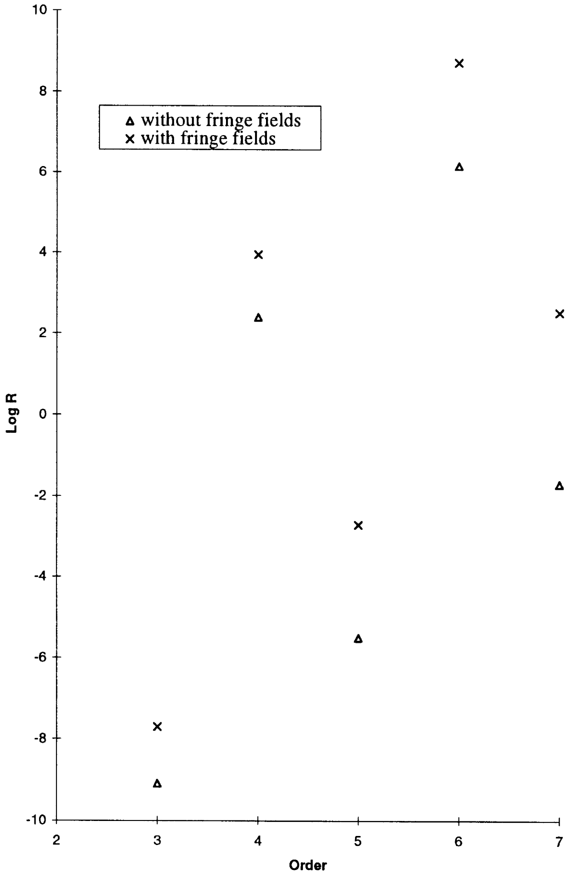

Fig. 4. Average absolute value of the resonance strengths for orders 3–7.

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